Reprezentarea grafica a functiilor reale

Pentru a trasa graficul unei functii , parcurgem mai multe etape :

1) ***Domeniul maxim de definitie***

a) gasirea domeniului maxim de definitie

b) Gf $\cap$ Ox => f(x)=0

c) Gf $\cap$ Oy => x=0 , f(x)= o valoare

d) $\lim_{x\to\pm\infty} f(x)$ (daca e constanta => y=k => asimptota orizontala)

2) ***Semnul functiei***

a) semnul functiei

b) paritatea functiei
f(x)=f(-x) => functia e simetrica fata de axa Oy
f(x)=-f(x) => functia e simetrica fata de origine

c) continuitatea functiei

d) periodicitatea

3) ***Asimptote***

a) orizontale

b) verticale

c) oblice

4) ***Derivata intai***

a) calculul derivatei intai

b) radacinile derivatei intai si valorile funtiei pe radacinile derivatei

c) tabelul

5) ***Derivata a doua***

a) calculul derivatei a doua

b) radacinile derivatei a doua si valorile functiei pe radacinile derivatei
   - determinarea punctelor de inflexiune , de maxim si minim local

c) semnul derivatei a doua

6) ***Tabelul de variatie al functiei***

X	
F’(x)	
F’’(x)	
F(x)	

7) ***Trasarea graficului***

- in grafic se incepe cu trasarea asimptotelor

Exemple:

f(x)= $x^4 - 8x^2$

1)a)f: **R**→**R**

b)f(x)=0

$x^4 - 8x^2 = 0$

$x^2(x^2 - 8) = 0$

$x_1 = x_2 = 0$

$x_3 = 2\sqrt{2}$

$x_4 = -2\sqrt{2}$

c)f(0)=0-0=0

d) $\lim_{x \to \pm\infty} f(x) = \pm\infty$

2)
a)

x	$-2\sqrt{2}$ 0 $2\sqrt{2}$
x^2	+++++++++++++++++++++++0++++++++++++++++++++++++++
$x^2 - 8$	++++0---0+++
f(x)	++++0-------------------0-----------------------0+++

b)f(x)=f(-x)

=> functie para
=> graficul este simetric fata de axa Ox

3) Asimptote nu exista

4) Derivata intai

$f'(x)=4x^3-16x=4x(x^2-4)$

$x_1=0$

$x_{2,3}=\pm 2$

f(0)=0
f(2)=-16
f(-2)=-16

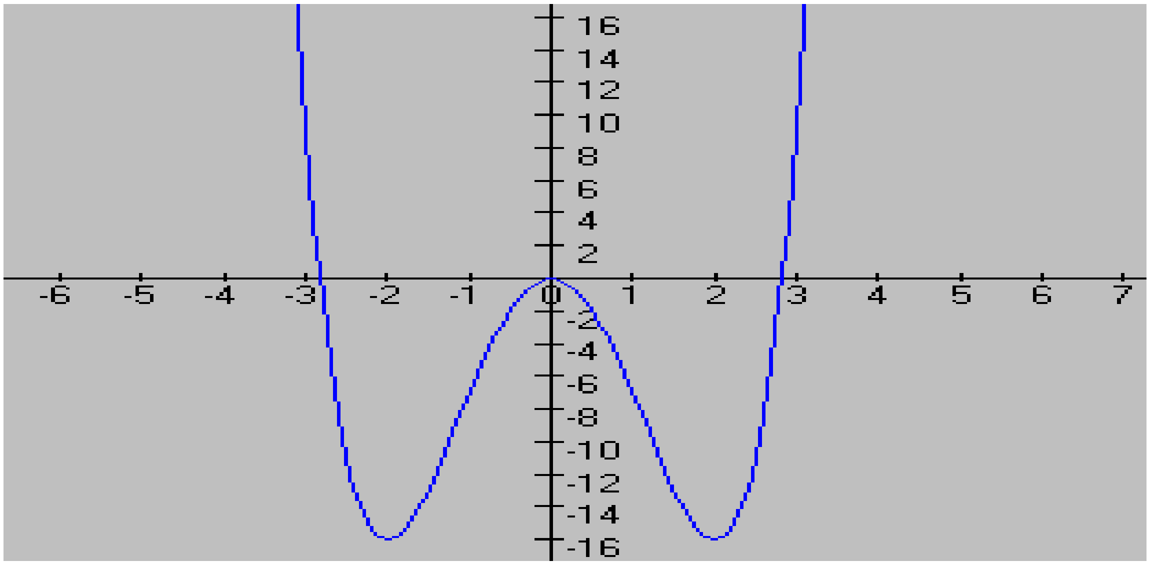
x	-∞ -2 0 2 +∞
x	----------------------0+++++++++++++++++++++++++
x^2-4	+++++++++++0--------------------0++++++++++++++
f'(x)	-----------0+++++++++++0---------0++++++++++++++

5) Derivata a doua

$f''(x)=12x^2-16$

$x_{1,2}=\pm\frac{2\sqrt{3}}{3}$ $f(\frac{2\sqrt{3}}{3})=-\frac{80}{9}$ $f(-\frac{2\sqrt{3}}{3})=-\frac{80}{9}$

X	-∞ $-2\sqrt{2}$ -2 $-\frac{2\sqrt{3}}{3}$ 0 $\frac{2\sqrt{3}}{3}$ 2 $2\sqrt{2}$ +∞
f'(x)	-------------0+++++++++++0-----------0+++++++++++++
f''(x)	+++++++++++++++++++0-----------0++++++++++++++++++
f(x)	-∞-----0----16+++ $-\frac{80}{9}$ ----0+++++ $-\frac{80}{9}$ ---16++++0++++ +∞ m i M i m



f(x)= $\frac{x^2-16}{x}$

1)a)f:**R**\{0}→**R**

b)f(x)=0

$x^4-16=0$

$x_1=4$ A(4,0) ; B(-4,0)

$x_2=-4$

c)f(0)= nu exista

d) $\lim_{x\to\pm\infty} f(x)=\pm\infty$

2)
a)

x	-∞ -4 0 4 +∞
x^2-16	+++++++++++++0-------------------------0+++++++++++++
x	-------------------------0+++++++++++++++++++++++++++
f(x)	-------------0+++++++++++|-------------0++++++++++++

x∈(-∞,-4)∪(0,4) => f(x)<0
x∈(-4,0)∪(4,+∞) => f(x)>0

b)f(x)=f(-x)
=> functie para
=> graficul este simetric fata de axa Ox

c)functia este continua pe **R**\{0}

3) Asimptote

y=x => asimptota oblica la ±∞

$\lim_{x \to 0} f(x) = \frac{x^2 - 16}{x} = \frac{-16}{0+} = \infty$ => x=0 asimptota verticala la ±∞

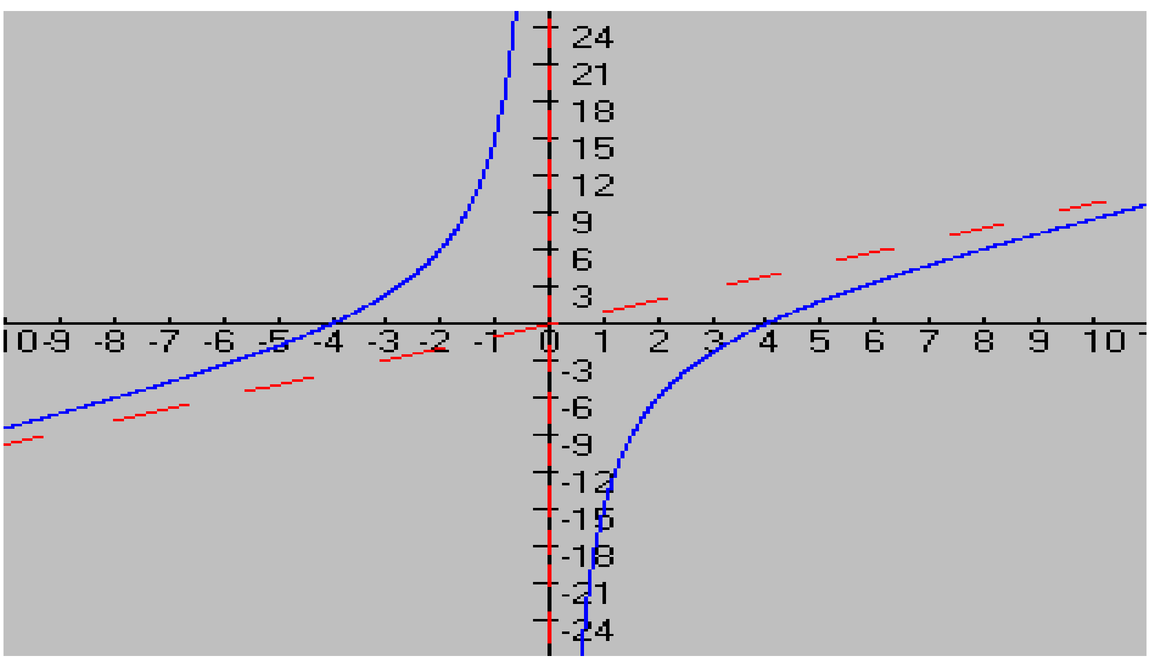
4) Derivata intai

f'(x)= $\frac{x^2 + 16}{x^2} > 0$

5) Derivata a doua

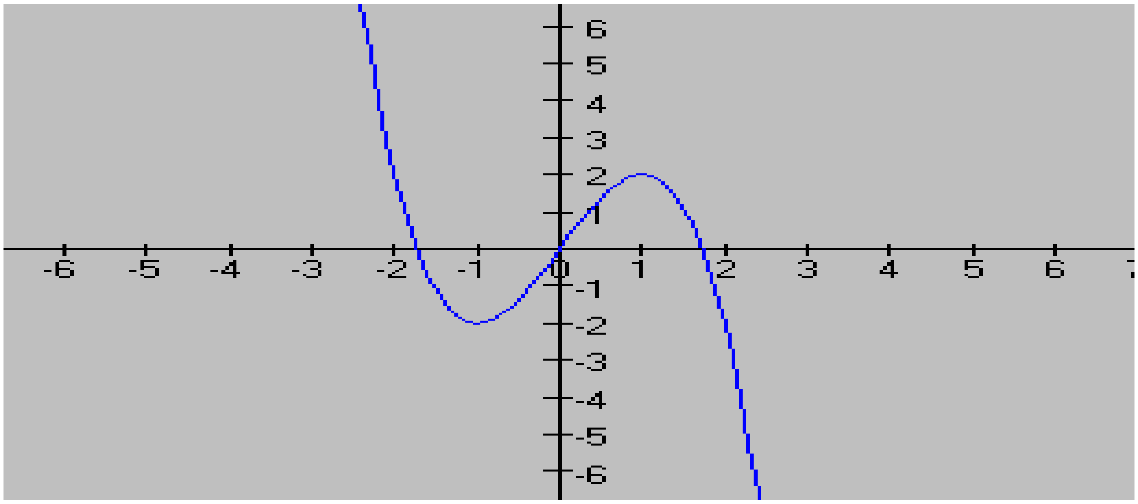
f''(x)= $\frac{x - 32}{x^3}$

X	-∞	0	+∞
f'(x)	+++++++++++++++++++++++++	|	+++++++++++++++++++++++++
f''(x)	+++++++++++++++++++++++++	|	-------------------------
f(x)	-∞ ↗	|	↗ +∞

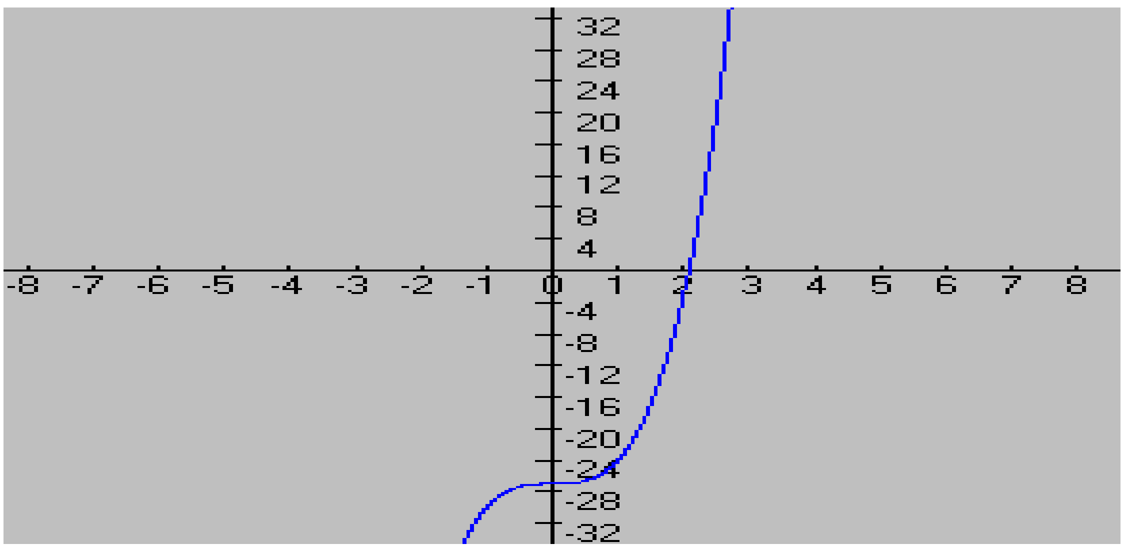


<u>Alte grafice de functii :</u>

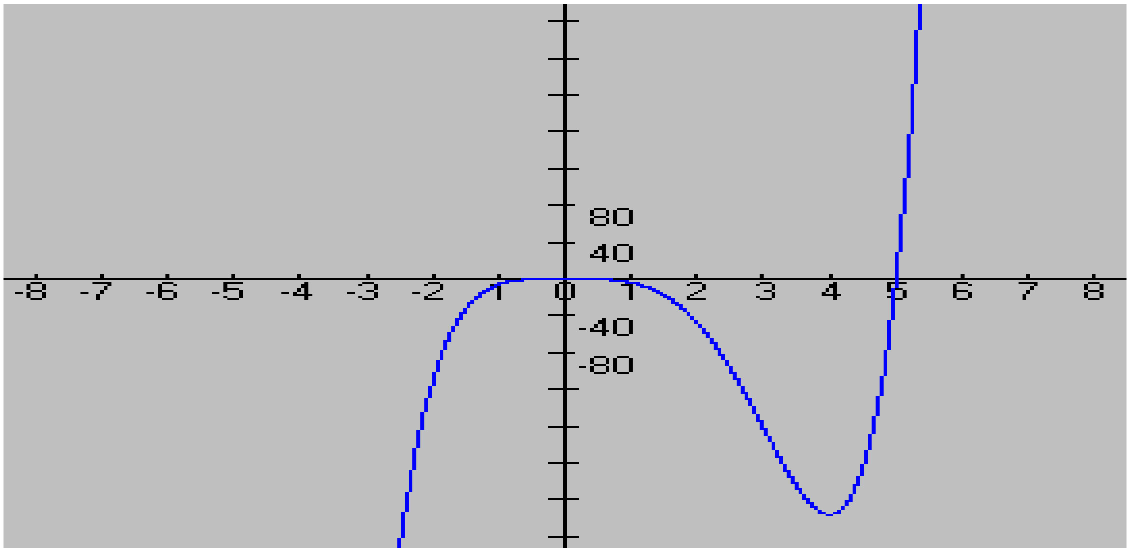
1)f(x)=$-x^3+3x$



2)f(x)=$3x^3-27$



3)f(x)=x^5-5x^4



4) $f(x)=\frac{2x}{x^2+1}$