METODA INDUCTIEI MATEMATICE COMPLETE. ANALIZA COMBINATORIE. BINOMUL LUI NEWTON. SUME.

1. METODA INDUCTIEI MATEMATICE COMPLETE

Este o metoda de rationament prin care stabilim ca:

O proprietate P(n) care depinde de un numar natural n este verificata pentru orice numar natural $n \geq k$ atunci sunt satisfacute simultan conditiile:

a) Proprietatea P(n) este adevarata pentru n=k; $k \in \boldsymbol{N}$

b) $(P(k), k \leq n) \Rightarrow P(n+1), (\forall)\ n \geq k$, adica presupunem P(k) adevarata pentru orice $k \leq n$ rezulta p(n+1) adevarata, pentru orice $n \geq k$.

2. PERMUTARI

Fie E={1, 2, …,n} o multime finita cu n elemente. Se numeste permutare a multimii E orice functie bijectiva $f : E \to E$.

Notam permutarea in felul urmator

$$f : \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ f(1) & f(2) & f(3) & \dots & f(n) \end{pmatrix}$$

Notam numarul de permutari Pn:	**Pn= n!=1.2.3…n**
conditie de existenta:	$n \in \boldsymbol{N}$
conventie:	0!=1 ; 1!=1
	Pn=n(n-1)!=n(n-1)(n-2)!

3. ARANJAMENTE

Notam cu $\mathbf{A_n^k}$

Sistemele ordonate cu k elemente, care se pot forma cu elementele unei multimi cu n elemente ($n \geq k$), se numesc aranjamente de n elemente luate cate k.

$$\mathbf{A_n^k = n!/(n-k)! = n(n-1)(n-2)\dots(n-k+1) = (n-k+1)A_n^{k-1}}$$

c.e. $n \geq k$

conventie: $n=k \Rightarrow \mathbf{A_n^n = Pn}$

4. COMBINARI C_n^k

$$C_n^k = \frac{A_n^k}{P_n} = \frac{n!}{(n-k)!k!} = \frac{n(n-1)\dots(n-k+1)}{1\cdot 2\cdot 3\cdot 4\dots k} = \frac{n-k+1}{k} C_n^{k-1}$$

<u>conventie</u>: $\mathbf{C_n^0 = C_n^n = 1}$ <u>c.e.</u> $n \geq k$

Formule pentru combinari complementare: $\mathbf{C_n^k = C_n^{n-k}}$

$\mathbf{C_n^k = C_{n-1}^k + C_{n-1}^{k-1}}$

5. BINOMUL LUI NEWTON

Daca a, b$\in$ ***R***, n$\in$ ***N***, atunci:

$$(a+b)^n=C_n^0a^n+C_n^1a^{n-1}b+C_n^2a^{n-2}b^2+\ldots+C_n^ka^{n-k}b^k+\ldots+C_n^{n-1}ab^{n-1}+C_n^nb^n$$

sau

$$(a+b)^n=\sum_{k=0}^{n} C_n^ka^{n-k}b^k=\sum_{k=0}^{n}T_{k+1},\ \text{unde } T_{k+1}=C_n^ka^{n-k}b^k$$

T_{k+1}=termen general

k=se numeste rangul termenului al dezvoltarii

$$(a-b)^n= C_n^0a^n-C_n^1a^{n-1}b+C_n^2a^{n-2}b^2-\ldots+(-1)^{n-k}C_n^ka^{n-k}b^k+\ldots+(-1)^{n-1}C_n^{n-1}ab^{n-1}+(-1)^nC_n^nb^n$$

sau

$$(a-b)^n=(-1)^k\sum_{k=0}^{n} C_n^ka^{n-k}b^k=\sum_{k=0}^{n}T_{k+1},\ \text{unde } T_{k+1}=(-1)^n C_n^ka^{n-k}b^k$$

Obs: 1) in dezvoltarea **(a+b)^n**, dupa formula lui Newton, sunt n+1 termeni.

2) C_n^0, C_n^1, C_n^2,…,C_n^n se numesc coeficienti binomiali

3) Sa se faca distinctie intre coeficientul unui termen al dezvoltarii si coeficientul binomial al aceluiasi termen.

4) Pentru a determina rangul celui mai mare termen folosim relatia:

$$\frac{T_{k+2}}{T_{k+1}}=\frac{n-k}{k+1}\cdot\frac{b}{a}\quad \text{si studiem doua cazuri:}$$

$$\frac{T_{k+2}}{T_{k+1}}\geq 1 \quad \text{si} \quad \frac{T_{k+2}}{T_{k+1}}<1$$

5) In dezvoltarea $(a+b)^n$ si $(a-b)^n$, daca a=b atunci:

$$C_n^0+C_n^1+C_n^2+\ldots+C_n^n=2^n$$

$$C_n^0+C_n^2+C_n^4+\ldots=C_n^1+C_n^3+C_n^5+\ldots=2^{n-1}$$

6) Identitatile utile:

a) $C_n^k=C_{n-1}^{k-1}+C_{n-2}^{k-1}+\ldots+C_{k-1}^{k-1}$

b) $C_{n+k}^k=C_n^0C_m^k+C_n^1C_m^{k-1}+\ldots+C_n^kC_m^0$

7) Suma puterilor asemenea ale primelor n numere naturale

Fie k≥1 un numar natural si $S_k=1^k+2^k+3^k+\ldots+n^k$

a) $S_1=1+2+3+\ldots+n=\sum_{k=1}^{n}k=\frac{n(n+1)}{2}$

Folosim dezvoltarea $(a+1)^2=a^2+2a+1$ pentru demonstratie unde a=1,2,…n.

b) $S_2=1^2+2^2+3^2+\ldots+n^2=\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}$

Folosim dezvoltarea $(a+1)^3=a^3+3a^2+3a+1$, pentru demonstratie, unde $a=1,2,\ldots n$.

c) $S_3=1^3+2^3+3^3+\ldots n^3=\sum_{k=1}^{n} k^3=[\frac{n(n+1)}{2}]^2$

Folosim dezvoltarea $(a+1)^4=a^4+4a^3+6a^2+4a+1$, pentru demonstratie, unde $a=1,2,\ldots n$

d) $S_k=\frac{(n+1)^{k+1}-(n+1)-\sum_{p=2}^{k} C_{k+1}^{p}\cdot S_{p-1}}{C_{k+1}^{1}}$

Caz particular

$S_4=\frac{n(n+1)(2n+1)(3n2+3n-1)}{30}$

6. PROGRESII ARITMETICE SI GEOMETRICE

a) PROGRESII ARITMETIVE ÷

Teorema : Fie numerele a_{n-1}, a_n, a_{n+1} in progresie aritmetica. Atunci:

$$\mathbf{2a_n=a_{n-1}+a_{n+1}}$$

Def: Fie numerele a_1, a_2, a_3,…,a_n in progresie aritmetica, daca $\mathbf{a_n=a_1+(n-1)r}$ sau $\mathbf{a_n=a_{n-1}+1}$, unde:
a_n= ultimul termen
a_1=primul termen
a_{n-1}=penultimul termen
n=numarul de termeni
r=ratia progresiei aritmetice

Obs: Pentru verificare $\mathbf{r=a_2-a_1=a_3-a_2=a_4-a_3=\ldots=a_n-a_{n-1}}$

P: $\mathbf{Sn=a_1+a_2+\ldots+a_n=\frac{(a_1+a_n)n}{2}=\frac{[2a_1+(n-1)r]n}{2}}$

b) PROGRESII GEOMETRICE ∺

Teorema: Fie numerele b_{n-1}, b_n, b_{n+1} in progresie geometrica. Atunci

$$\mathbf{b_n^2=b_{n-1}\cdot b_{n+1}}$$

Def: Fie numerele b_1, b_2,…b_n in progresie geometrica, daca $\mathbf{b_n=b_1 \cdot q^n}$ sau $\mathbf{b_n=b_{n-1} \cdot q}$ unde: b_n=ultimul termen

b_1=primul termen
b_{n-1}=penultimul termen
n=numarul de termeni
q=ratia progresiei geometrice

Obs: pentru verificare $\mathbf{q=a_2/a_1=a_3/a_2=\ldots=a_n/a_{n-1}}$

$$\mathbf{P:\ Sn=b_1(1+q+q^2+q^3+\ldots+q^{n-1})=b_1 \cdot \frac{q^n-1}{q-1}=b_1+b_2+\ldots+b_n}$$