Graficele functiilor trigonometrice

În trasarea graficelor functiiolr trigonometrice se urmaresc mai multe etape:

I

a) gasirea domeniul maxim de definitie a functiei
b) gasirea intersectiei graficului cu axa Ox (f(x)=0)
c) gasirea intersectiei graficului cu axa Oy (se calculeaza f(0))
d) se calculeaza $\lim_{x \to \pm\infty} f(x)$

II

a) se studiaza paritatea sau imparitatea functiei
b) se studiaza periodicitatea functiei
c) se studiaza continuitatea functiei
d) se studiaza semnul functiei pe domeniul de definitie

III

a) se cauta asimptota orizontala
b) se cauta asimptota oblica
c) se cauta asimptota verticala în punctele de acumulare unde functia nu este definita

IV

a) se calculeaza derivata I
b) se gasesc radacinile derivatei I si valoarea functiei în aceste radacini
c) se gaseste semnul derivatei I

V

a) se calculeaza derivata II
b) se gasesc radacinile derivatei II si valoarea functiei în aceste radacini
c) se gaseste semnul derivatei II

VI

a) se construieste tabelul de variatie a functiilor

VII

a) se traseaza graficul functiei

Sa se reprezinte grafic functiile:

i) $f(x) = \sin x\,(1+\cos x)$

1

$f_1(x) = \sin x \quad f_1(x+2k\pi) = f_1(x) \Rightarrow T = 2k\pi \Rightarrow T_0 = 2\pi$

$f_2(x) = 1+\cos x \quad f_2(x+2k\pi) = f_2(x) \Rightarrow T = 2k\pi \Rightarrow T_0 = 2\pi$

$\Rightarrow$ putem considera restrictia $f/_{[0,2\pi]} \to R$

$f(x) = 0 \Rightarrow \sin x\,(1+\cos x) = 0$

$\sin x = 0 \qquad 1+\cos x = 0$

$x_{1,2,3} = 0, \pi, 2\pi \qquad \cos x = -1 \Rightarrow x_4 = \pi$

2

$f(-x) = \sin(-x)\,(1+\cos(-x)) = -\sin x\,(1+\cos x) = -f(x)$

$\Rightarrow f(x) =$ functie impara $\Rightarrow$ graficul simetric fata de origine

$f(x+2k\pi) = f(x) \Rightarrow f$ periodica de $T_0 = 2\pi$

functia e continua pe domeniul de definitie ca produs de functii elementare

3

- asimptote orizontale

$\lim\limits_{x\to\pm\infty} f(x) = \sin x(1+\cos x) = (\lim\limits_{x\to\pm\infty} \sin x)(1+\lim\limits_{x\to\pm\infty} \cos x) = -$

$\Rightarrow (\nexists)$ asimptote orizontale

- asimptote oblice

$y = mx + n$

$m = \lim\limits_{x\to\pm\infty} \frac{f(x)}{x} = \lim\limits_{x\to\pm\infty} \frac{\sin x}{x}(1+\cos x) = -$

$\Rightarrow (\nexists)$ asimptote oblice

$-$ asimptote verticale - ($(\nexists)$ pct. de acumulare unde functia nu este definita)

4

$f^1(x) = (\sin x(1+\cos x))' = \cos x(1+\cos x) - \sin^2 x = \cos x + \cos^2 x - \sin^2 x =$

$= \cos x + \cos^2 x - 1 + \cos^2 x = 2\cos^2 x + \cos x - 1$

$f^1(x) = 0$

$\Rightarrow 2\cos^2 x + \cos x - 1 = 0 \qquad \cos x = t \quad t \in [-1,1]$

$2t^2 + t - 1 = 0$

$\Delta = 1 + 8 = 9$

$t_{1,2} = \frac{-1 \pm 3}{4} \quad \to t_1 = -1 \quad t_2 = \frac{1}{2}$

$\Rightarrow (\cos x + 1)(\cos x - \frac{1}{2}) = 0$

$\cos x = -1 \qquad \cos x = \frac{1}{2}$

$x_1 = \pi \quad f(\pi) = 0 \qquad x_{1,2} = \frac{\pi}{3}, \frac{5\pi}{3} \quad f(\frac{\pi}{3}) = \frac{3\sqrt{3}}{4}; f(\frac{5\pi}{3}) = -\frac{3\sqrt{3}}{4}$

x		π/3		π		5π/3	
cosx+1	+ +		+ +	0	+ + +		+ +
cosx+	+ +	0	- - -		- - - -	0	+ + +
f'(x)	+ +	0	- - - -	0	- - - -	0	+ + +

5

$f^2(x) = (2\cos^2 x + \cos x - 1)' = -4\cos x \sin x - \sin x = -\sin x(4\cos x + 1)$

$f^2(x) = 0$

$\sin x = 0 \qquad \cos x = -\frac{1}{4}$

$x_{1,2,3} = 0, \pi, 2\pi \qquad x_4 = \pi - \arccos\frac{1}{4} \;; f(\pi - \arccos\frac{1}{4}) = \frac{3\sqrt{15}}{16}$

$x_5 = \pi + \arccos\frac{1}{4} \;; f(\pi + \arccos\frac{1}{4}) = \frac{-3\sqrt{15}}{16}$

x	0	π- arccos	π	π+ arccos	2π

-sin x	0 - - - - - - - - - 0 + + + + + 0
4cosx+1	+ + + 0 - - - - - - - 0 + + + + +
f''(x)	0 - - - - 0 + + + + 0 - - - - 0 + + + + + 0

VI

x	0	π/3	π- arccos	π	π+ arccos	5π/3	2π
f'(x)	+ + + 0 - - - - - - - 0 - - - - - - 0 + + +						
f''(x)	0 - - - - - 0 + + 0 - - - 0 + + + + + + 0						
f(x)							

ii) $f(x) = \cos 2x + 2\cos x + 1$

1

$f_1(x) = \cos 2x \quad f_1(x + 2k\pi) = f_1(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$f_2(x) = \cos x \quad f_1(x + 2k\pi) = f_1(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$\Rightarrow$ putem considera restrictia $f/_{[0,2\pi]} \to R$

$f(x) = 0 \ \Rightarrow \cos 2x + 2\cos x + 1 = 0$

$2\cos^2 x - 1 + 2\cos x + 1 = 0$

$2(\cos^2 x + \cos x) = 0 \ \Big| : 2$

$\cos x(\cos x + 1) = 0$

$\cos x = 0 \qquad x_1 = \frac{\pi}{2} \quad x_2 = \frac{3\pi}{2}$

$\cos x = -1 \qquad x_3 = \pi$

2

$f(-x) = \cos(-2x) + \cos(-x) + 1 = \cos 2x + 2\cos x + 1 = f(x)$

$\Rightarrow f(x)$ functie para $\Rightarrow$ grafic simetric fata de Oy

$f(x + 2k\pi) = f(x) \ \Rightarrow$ f periodica de perioada principala 2π

functia e continua pe domeniul de definitie ca produs de functii continue

3

asimptote orizontale :

$\lim_{x\to\pm\infty} \cos 2x + \cos x + 1 = \lim_{x\to\pm\infty} \cos 2x + 2\lim_{x\to\pm\infty} \cos x + 1 = -$

$\Rightarrow$ nu exista asimptote orizontale

asimptote oblice :

$y = mx + n$

$m = \lim_{x\to\pm\infty} \frac{\cos 2x}{x} + 2\lim_{x\to\pm\infty} \frac{\cos x}{x} + 0 = -$

$\Rightarrow$ nu exista asimptote oblice

asimptote verticale :

nu exista pcte de acumulare unde functia nu e definita

$\Rightarrow$ nu exista asimptote orizontale

4

$$f^1(x) = (\cos 2x + 2\cos x + 1)' = -2\sin 2x - 2\sin x = -4\sin x\cos x - 2\sin x =$$
$$= -2\sin x(2\cos x + 1)$$

$$f^1(x) = 0 \;\Rightarrow \sin x = 0 \qquad x_1 = 0; x_2 = \pi; x_3 = 2\pi$$

$$f(0) = 4; f(\pi) = 0; f(2\pi) = 4$$

$$\Rightarrow \cos x = -\frac{1}{2} \qquad x_1 = \frac{2\pi}{3} \quad ; \quad x_2 = \frac{4\pi}{3}$$

$$f(\frac{2\pi}{3}) = -\frac{1}{2}; f(\frac{4\pi}{3}) = -\frac{1}{2}$$

x	0 2π/3 π 4π/3 2π
-2sinx	0- - - - - 0 + + + + + +
2cosx+1	+ + + 0 - - - - 0 + + + + +
$f^1(x)$	0 - - - - 0 + + + 0 - - - 0 + + + + + + +

5

$$f^2(x) = (-2\sin 2x - 2\sin x)' = -4\cos 2x - 2\cos x = -8\cos^2 x + 4 - 2\cos x$$
$$= -2(4\cos^2 x + \cos x - 2)$$

$$f^2(x) = 0$$

$$4\cos^2 x + \cos x - 2 = 0$$

$$\Delta = 1 + 32 = 33$$

$$\cos x_1 = \frac{-1+\sqrt{33}}{8} = \frac{\sqrt{33}-1}{8} \Rightarrow x_1 = \arccos\frac{\sqrt{33}-1}{8} \approx 0.93\,; x_1 \in (\frac{\pi}{4}, \frac{\pi}{3})$$

$$x_2 = 2\pi - \arccos\frac{\sqrt{33}-1}{8} \approx 5.34; x_2 \in (\frac{5\pi}{3}, \frac{7\pi}{4})$$

$$\cos x_2 = \frac{-1-\sqrt{33}}{8} = -\frac{\sqrt{33}+1}{8} \Rightarrow x_3 = \pi - \arccos\frac{\sqrt{33}+1}{8} \approx 2.57; x_3 \in (\frac{5\pi}{7}, \frac{5\pi}{6})$$

$$x_4 = \pi + \arccos\frac{\sqrt{33}+1}{8} \approx 3.70; x_4 \in (\frac{9\pi}{7}, \frac{8\pi}{7})$$

6

x	0	x_1	2π/3	x_3	π	x_4	4π/3	x_2	2π
$f^1(x)$	0 - - - - - - - - 0 + + + + 0 - - - - - - - 0 + + + + + 0								
$f^2(x)$	- - - - -0 + + + + + 0 - - - - -0 + + + + + + 0 - - - -								
f(x)	4	1.88	-0.5	-0.26	0	-0.26	-0.5	1.88	4

iii) $f(x) = \sin x \cos^3 x$

$f_1(x) = \cos^3 x \quad f_1(x+2k\pi) = f_1(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$f_2(x) = \sin x \quad f_2(x+2k\pi) = f_2(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$\Rightarrow$ putem considera restrictia $f/_{[0,2\pi]} \to R$

$f(x) = 0 \ \Rightarrow \sin x \cos^3 x = 0$

$\sin x = 0 \ \Rightarrow x_1 = 0 \quad x_2 = \pi \quad x_3 = 2\pi$

$\cos^3 x = 0 \ \Rightarrow \ x_4 = \frac{\pi}{2} \quad x_5 = \frac{3\pi}{2}$

2

$f(-x) = \sin(-x)\cos^3(-x) = -\sin x \cos^3 x = -f(x)$

$\Rightarrow f(x)$ functie impara $\Rightarrow$ grafic simetric fata de origine

$f(x+2k\pi) = f(x) \ \Rightarrow$ f periodica de perioada principala 2π

functia e continua pe domeniul de definitie ca produs de functii continue

3

asimptote orizontale :

$\lim_{x\to\pm\infty} \sin x \cos^3 x = \lim_{x\to\pm\infty} \sin x \lim_{x\to\pm\infty} \cos^3 x = -$

$\Rightarrow$ nu exista asimptote orizontale

asimptote oblice :

$y = mx + n$

$m = \lim_{x\to\pm\infty} \frac{\sin x}{x} \lim_{x\to\pm\infty} \cos^3 x = -$

$\Rightarrow$ nu exista asimptote oblice

asimptote verticale :

nu exista pcte de acumulare unde functia nu e definita

$\Rightarrow$ nu exista asimptote orizontale

4

$f^1(x) = (\sin x \cos^3 x)' = \cos^4 x - 3\sin x \cos^2 \sin x = \cos^2 x(\cos^2 x - 3\sin^2 x)$

$f^1(x) = 0$

$$\cos^2 x = 0 \Rightarrow x_1 = \frac{\pi}{2} \quad f(x_1) = 0; x_2 = \frac{3\pi}{2} \quad f(x_2) = 0$$

$$\cos^2 x - 3\sin^2 x = 0 \Rightarrow \cos^2 x = 3\sin^2 x \,\Big|\, : 3\cos^2 x$$

$$\Rightarrow tg^2 x = \frac{1}{3} \quad \Rightarrow tgx = \pm\frac{\sqrt{3}}{3}$$

$$x_3 = arctg\frac{\sqrt{3}}{3} = \frac{\pi}{6} \qquad f(\frac{\pi}{6}) = \frac{\sqrt{27}}{16}$$

$$x_4 = \pi + arctg\frac{\sqrt{3}}{3} = \frac{7\pi}{6} \qquad f(\frac{7\pi}{6}) = \frac{\sqrt{27}}{16}$$

$$x_5 = \pi - arctg\frac{\sqrt{3}}{3} = \frac{5\pi}{6} \qquad f(\frac{5\pi}{6}) = -\frac{\sqrt{27}}{16}$$

$$x_6 = 2\pi - arctg\frac{\sqrt{3}}{3} = \frac{11\pi}{6} \qquad f(\frac{11\pi}{6}) = -\frac{\sqrt{27}}{16}$$

x	0 π/2 5π/6 7π/6 3π/2 11π/6 2π
$\cos^2 x - 3\sin^2 x$	+ + + + 0 - - - 0 + + 0 - - - - - - - 0 + + + + +
$\cos^2 x$	+ + + 0 + + + + + + + + 0 + + + + + +
$f^1(x)$	0 + + 0 - - - -0 + + +0 - - - 0 - - - 0 + + + + +

5

$$f^2(x) = [\cos^2 x(\cos^2 x - 3\sin^2 x)]' =$$

$$-2\cos x \sin x(\cos^2 x - 3\sin^2 x) + \cos^2 x(-2\cos x \sin x - 6\sin x \cos x) =$$

$$-2\cos^3 x \sin x + 6\sin^3 x \cos x - 2\cos^3 x \sin x - 6\sin x \cos^3 x$$

$$= -10\cos^3 x \sin x + 6\sin^3 x \cos x = -\sin x \cos x(10\cos^2 x - 6\sin^2 x)$$

$$f^2(x) = 0 \Rightarrow -\sin x \cos x = 0 \Rightarrow x_{1,2,3,4,5} = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

$$10\cos^2 x - 6\sin^2 x = 0 \Rightarrow 10\cos^2 x = 6\sin^2 x \,\Big|\, : 6\cos^2 x$$

$$\Rightarrow tg^2 x = \frac{5}{3} \Rightarrow tgx = \pm\sqrt{\frac{5}{3}} \Rightarrow$$

$$\Rightarrow x_6 = arctg\sqrt{\frac{5}{3}} \qquad f(x_6) \approx 0.18$$

$$x_7 = \pi - arctg\sqrt{\frac{5}{3}} \qquad f(x_7) \approx -0.18$$

$$x_8 = \pi + \operatorname{arctg}\sqrt{\frac{5}{3}} \qquad f(x_8) \approx 0.18$$

$$x_9 = 2\pi - \operatorname{arctg}\sqrt{\frac{5}{3}} \qquad f(x_9) \approx -0.18$$

x	0 π/2 π 3π/2 2π
sinx	0 + + + + + + + +0 - - - - - - - - - 0
cosx	+ + + + 0 - - - - - - - 0 + + + +
-sinxcosx	0 - - - - 0 + + + 0 - - - - 0 + + + 0

x	0 0.91 π/2 2.22 π 4.05 3π/2 5.37 2π
-sinxcosx	0 - - - 0 + + + 0 - - - - 0 + + + 0
10cos²-sin²x	+ + 0 - - - - 0 + + + + 0 - - - - - 0 + + +
$f^2(x)$	0 - - 0 + +0 - 0 + + 0 - - 0 + + + 0 - - 0 + + 0

6

x	0 π/6 0.91 π/2 2.22 5π/6 π 7π/6 4.05 3π/2 5.37 11π/6 2π
$f^1(x)$	0 + 0 - - - 0 - - - - 0 + + + 0 - - - - 0 - - - - 0 + +
$f^2(x)$	0 - - - 0 + +0 - - 0 + + + + 0 - - - - - 0 + + 0 - - 0 + + + + 0
f(x)	0 0.32 0.18 0 -0.18 -0.32 0 0.32 0.18 0 -0.18 -0.32 0

Se obseva ca graficul functiei pe intervalul [0, π] este identic cu cel pe intervalul (π,2π], în consecinta vom reprezenta functia doar pe intervalul [0, π].

iv) $f(x)=\dfrac{1-\cos^2 x}{(\sin x+\cos x)(\sin x-\cos x)}$

1

$$f(x)=\frac{1-\cos^2 x}{(\sin x+\cos x)(\sin x-\cos x)}=\frac{\sin^2 x}{\cos^2 x-\sin^2 x}=\frac{\sin^2 x}{\cos 2x}$$

$$\Rightarrow \cos 2x\neq 0\Rightarrow 2x\neq\frac{\pi}{2}+k\pi\,\Big|:2\Rightarrow x\neq\frac{\pi}{4}+\frac{k\pi}{2}$$

$$f(x+k\pi)=\frac{\sin^2(x+k\pi)}{\cos(2x+2k\pi)}=\frac{(-\sin x)^2}{\cos(2x)}=\frac{\sin^2 x}{\cos 2x}=f(x)$$

$$\Rightarrow T=k\pi\Rightarrow T_0=\pi$$

$\Rightarrow$ putem considera $f:[0,\pi]\setminus\{\frac{\pi}{4},\frac{3\pi}{4}\}\to R$

2

$f(-x)=\dfrac{\sin^2(-x)}{\cos(-2x)}=\dfrac{\sin^2 x}{\cos 2x}=f(x)\Rightarrow$ functie para $\Rightarrow$ graf. simetric fata de Oy

$f(x+k\pi)=f(x)\Rightarrow$ functie periodica de $T=k\pi$ $(T_0=\pi)$

functia e continua pe domeniul de definitie ca raport de functii continue

3

asimptote orizontale : -

$$\lim_{x\to\pm\infty}\frac{\sin^2 x}{\cos 2x}=-$$

asimptote oblice : -

$y=mx+n$

$$m=\lim_{x\to\pm\infty}\frac{\sin^2 x}{x}\cdot\frac{1}{\cos 2x}=\lim_{x\to\pm\infty}\frac{\sin^2 x}{x}\cdot\lim_{x\to\pm\infty}\frac{1}{\cos 2x}=-$$

asimptote verticale :

$$l_s=\lim_{\substack{x\to\frac{\pi}{4}\\ x<\frac{\pi}{4}}}f(x)=\frac{\frac{1}{2}}{0_+}=+\infty$$

$$l_d = \lim_{\substack{x \to \frac{\pi}{4} \\ x > \frac{\pi}{4}}} f(x) = \frac{\frac{1}{2}}{0_-} = -\infty$$

$$\Rightarrow x = \frac{\pi}{4} \text{ asimptota verticala}$$

4

$$f^1(x) = \frac{2\sin x \cos x \cos 2x + 2\sin 2x \sin^2 x}{\cos^2 2x} = \frac{\sin 2x(\cos 2x + 2\sin^2 x)}{\cos^2 2x} =$$

$$\frac{\sin 2x(\cos^2 x - \sin^2 x + 2\sin^2 x)}{\cos^2 2x} = \frac{\sin 2x(\cos^2 x + \sin^2 x)}{\cos^2 2x} = \frac{\sin 2x}{\cos^2 2x}$$

$$f^1(x) = 0 \Rightarrow x_{1,2,3} = 0, \frac{\pi}{2}, \pi$$

$$\Rightarrow \text{pt } x \in (0, \frac{\pi}{2}) \setminus \{\frac{\pi}{4}\}, f^1(x) > 0 \quad ; \text{pt } x \in (\frac{\pi}{2}, \pi) \setminus \{\frac{3\pi}{4}\}, f^1(x) < 0$$

5

$$f^2(x) = \frac{\sin 2x}{\cos^2 2x} = \frac{2\cos^3 2x + 4\cos 2x \sin^2 2x}{\cos^4 2x} = \frac{2\cos^2 2x + 4\sin^2 2x}{\cos^3 2x} =$$

$$\frac{2\cos^2 2x + 2\sin^2 2x + 2\sin^2 2x}{\cos^3 2x} = \frac{1 + 2\sin^2 2x}{\cos^3 2x}$$

$$f^2(x) > 0 \text{ pt } x \in (0, \frac{\pi}{4}) \cup (\frac{3\pi}{4}, \pi)$$

$$f^2(x) < 0 \text{ pt } x \in (\frac{\pi}{4}, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{4})$$

6

x	0		π/4		π/2		3π/4		π
$f^1(x)$	0	+ + +	I	+ + + +	0	- - - - -	I	- - - - - -	0
$f^2(x)$	0	+ + + +	I	- - - - - -	-	- - - - - -	I	+ + + + + +	
f(x)	0		$^{+\infty}I_{-\infty}$		-1		$_{-\infty}I^{+\infty}$		0

v) $f(x) = \frac{\cos x}{\cos 2x}$

$f_1(x) = \cos x \quad f_1(x + 2k\pi) = f_1(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$f_2(x) = \cos 2x \quad f_2(x + 2k\pi) = f_2(x) \quad \Rightarrow T = 2k\pi \ \Rightarrow T_0 = 2\pi$

$\cos 2x \neq 0 \Rightarrow 2x \neq \frac{\pi}{2} + k\pi \,\Big|\, : 2 \Rightarrow x \neq \frac{\pi}{4} + \frac{k\pi}{2}$

$\Rightarrow$ putem considera $f : [0,2\pi] \setminus \{\frac{\pi}{4} + \frac{k\pi}{2}\} \to R$

$f(x) = 0 \Rightarrow \cos x = 0 \quad \Rightarrow \quad x_1 = \frac{\pi}{2} \quad x_2 = \frac{3\pi}{2}$

2

$f(-x) = \frac{\cos(-x)}{\cos(-2x)} = \frac{\cos x}{\cos 2x} = f(x)$

$\Rightarrow f(x)$ functie para $\Rightarrow$ grafic simetric fata de Oy

$f(x + 2k\pi) = f(x) \quad \Rightarrow$ f periodica de perioada principala 2π

functia e continua pe domeniul de definitie ca raport de functii continue

3

asimptote orizontale :

$\lim_{x \to \pm\infty} \frac{\cos x}{\cos 2x} = -$

$\Rightarrow$ nu exista asimptote orizontale

asimptote oblice :

$y = mx + n$

$m = \lim_{x \to \pm\infty} \frac{\cos x}{x} \lim_{x \to \pm\infty} \frac{1}{\cos 2x} = -$

$\Rightarrow$ nu exista asimptote oblice

asimptote verticale :

$\lim_{\substack{x \to \frac{\pi}{4} \\ x < \frac{\pi}{4}}} f(x) = \frac{\frac{\sqrt{2}}{2}}{0_+} = +\infty \quad ; \quad \lim_{\substack{x \to \frac{\pi}{4} \\ x > \frac{\pi}{4}}} f(x) = \frac{\frac{\sqrt{2}}{2}}{0_-} = -\infty \Rightarrow \frac{\pi}{4} =$ asimptota verticala

$$\lim_{\substack{x\to\frac{3\pi}{4}\\ x<\frac{3\pi}{4}}} f(x)=\frac{-\frac{\sqrt{2}}{2}}{0_-}=+\infty \quad ; \quad \lim_{\substack{x\to\frac{3\pi}{4}\\ x>\frac{3\pi}{4}}} f(x)=\frac{-\frac{\sqrt{2}}{2}}{0_+}=-\infty \Rightarrow \frac{3\pi}{4}=\text{asimptota verticala}$$

$$\lim_{\substack{x\to\frac{5\pi}{4}\\ x<\frac{5\pi}{4}}} f(x)=\frac{-\frac{\sqrt{2}}{2}}{0_+}=-\infty \quad ; \quad \lim_{\substack{x\to\frac{3\pi}{4}\\ x>\frac{3\pi}{4}}} f(x)=\frac{-\frac{\sqrt{2}}{2}}{0_-}=+\infty \Rightarrow \frac{5\pi}{4}=\text{asimptota verticala}$$

$$\lim_{\substack{x\to\frac{7\pi}{4}\\ x<\frac{7\pi}{4}}} f(x)=\frac{\frac{\sqrt{2}}{2}}{0_-}=-\infty \quad ; \quad \lim_{\substack{x\to\frac{3\pi}{4}\\ x>\frac{3\pi}{4}}} f(x)=\frac{\frac{\sqrt{2}}{2}}{0_+}=+\infty \Rightarrow \frac{7\pi}{4}=\text{asimptota verticala}$$

4

$$f^1(x)=(\frac{\cos x}{\cos 2x})'=\frac{-\sin x\cos 2x+2\cos x\sin 2x}{\cos^2 2x}=$$

$$\frac{-\sin x(2\cos^2 x-1)+4\cos^2 x\sin x}{\cos^2 2x}=\frac{\sin x(1-2\cos^2 x+4\cos^2 x)}{\cos^2 2x}=$$

$$f^1(x)=\frac{\sin x(1+2\cos^2 x)}{\cos^2 2x}$$

$$f^1(x)=0 \qquad 1+2\cos^2 x>0 \Rightarrow \sin x=0 \Rightarrow x_{1,2,3}=0,\pi,2\pi$$

$$\left.\begin{array}{l}1+2\cos^2 x>0\\ \cos^2 x>0\end{array}\right\} \Rightarrow x\in[0,\pi]\setminus\{\frac{\pi}{4}+\frac{k\pi}{2}\}\ \ f^1(x)\geq 0; x\in(\pi,2\pi]\setminus\{\frac{\pi}{4}+\frac{k\pi}{2}\}\ \ f^1(x)<0$$

5

$$f^2(x)=(\frac{\sin x(1+2\cos^2 x)}{\cos^2 2x})'=$$

$$\frac{[\cos x(2\cos^2 x+1)-2\sin x(\cos x\sin x)]\cos^2 2x+2\sin x(2\cos^2 x+1)\sin 2x\cos 2x}{\cos^4 2x}=$$

$$=\frac{[2\cos^3 x+\cos x-2\sin^2 x\cos x]\cos^2 2x+2\sin x(2\cos^2 x+1)\sin 2x\cos 2x}{\cos^4 2x}=$$

$$= \frac{\cos x(2\cos^2 x + 1 - 2\sin^2 x)\cos 2x + 2\sin x(2\cos^2 x + 1)\sin 2x}{\cos^4 2x} =$$

$$= \frac{\cos x(2 + \cos 2x)\cos 2x + 2\sin x(2\cos^2 x + 1)\sin 2x}{\cos^4 2x} =$$

$$= \frac{\cos x(2 + \cos 2x)\cos 2x + 4\sin x(\cos 2x + 2)\sin x \cos x}{\cos^4 2x} =$$

$$\frac{\cos x(2 + \cos 2x)(\cos 2x + 4\sin^2 x)}{\cos^4 2x} =$$

$f^2(x) = 0;$

$2 + \cos x > 0 \ \ (\forall) \in D$

$\cos 2x + 4\sin^2 x = \cos^2 x + 3\sin^2 x > 0 \ \ (\forall)x \in D$

$$\Rightarrow \cos x = 0 \Rightarrow x_{1,2} = \frac{\pi}{2}, \frac{3\pi}{2}$$

x	0 π/4 π /2 3π/4 5π/4 3π/2 7π/4 2π
cosx	+ + + + 0 - - - - - - - - - 0 + + + + + +
cos³2x	+ + 0 - - - - - - - - 0 + + + 0 - - - - - - - - - 0 + + +
r	+ + +I - - - - 0 + + + I - - - - - I + + + +0 - - - - - I + + +

6

x	0 π/4 π /2 3π/4 π 5π/4 3π/2 7π/4 2π
$f^1(x)$	0 + + I + + + + + + I + + 0 - - - I - - - - - - - - - I - - - - 0
$f^2(x)$	+ + + I - - - - 0 + + + I - - - - - - -I + + + +0 - - - - - I + + +
f(x)	1 $^{+\infty}I_{-\infty}$ 0 $^{+\infty}I_{-\infty}$ -1 $^{-\infty}I_{+\infty}$ 0 $^{-\infty}I_{+\infty}$

vi) $f(x)=\frac{1-\sin x}{1-\cos x}$

$\cos x \neq 1 \Rightarrow f: R \setminus \bigcup_{k\in Z} 2k\pi \to R$

$f_1(x)=\cos 2x \quad f_1(x+2k\pi)=f_1(x) \quad \Rightarrow T=2k\pi \;\Rightarrow T_0=2\pi$

$f_2(x)=\cos x \quad f_1(x+2k\pi)=f_1(x) \quad \Rightarrow T=2k\pi \;\Rightarrow T_0=2\pi$

$f(x)=0 \;\Rightarrow \sin x=1 \Rightarrow x_k=\frac{\pi}{2}+k\pi$

2

$f(-x)=\frac{1+\sin x}{1-\cos x}$ functie oarecare

$f(x+2k\pi)=f(x) \;\Rightarrow$ f periodica de perioada principala $2\pi \Rightarrow f:(0,2\pi)\to R$

functia e continua pe domeniul de definitie ca produs de functii continue

3

asimptote orizontale :

$$\lim_{x\to\pm\infty}\frac{1+\sin x}{1-\cos x}=-$$

$\Rightarrow$ nu exista asimptote orizontale

asimptote oblice :

$y=mx+n$

$$m=\lim_{x\to\pm\infty}\frac{1-\sin x}{x}\cdot\lim_{x\to\pm\infty}\frac{1}{1-\cos x}=-$$

$\Rightarrow$ nu exista asimptote oblice

asimptote verticale :

$$\lim_{\substack{x\to 0\\ x>0}}\frac{1-\sin x}{1-\cos x}=\frac{1}{0_+}=+\infty \;;\; \lim_{\substack{x\to 2\pi\\ x<2\pi}}\frac{1-\sin x}{1-\cos x}=\frac{1}{0_+}=+\infty$$

$\Rightarrow x=0$ si $x=2\pi$ asimptote verticale

4

$$f^1(x)=\frac{\cos x(1-\cos x)+\sin x(1-\sin x)}{(1-\cos x)^2}=\frac{\cos x-\cos^2 x+\sin x-\sin^2 x}{(1-\cos x)^2}=$$

$$=\frac{1-(\sin x+\cos x)}{(1-\cos x)^2}=\frac{1-\sqrt{2}(\cos x\frac{\sqrt{2}}{2}+\sin x\frac{\sqrt{2}}{2})}{(1-\cos x)^2}=\frac{1-\sqrt{2}(\cos x-\frac{\pi}{4})}{(1-\cos x)^2}$$

$$f^1(x) = 0 \Rightarrow \cos(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{2} \Rightarrow x - \frac{\pi}{4} = \frac{\pi}{4} \Rightarrow x = \frac{\pi}{2}$$

$$x - \frac{\pi}{4} = 2\pi - \frac{\pi}{4} \Rightarrow x = 2\pi$$

$$x = 2\pi \notin D \Rightarrow x = \frac{\pi}{2}$$

6

x	0	π/4	π /2	3π/4	π	5π/4	3π/2	7π/4	2π
$f^1(x)$	- -	- -	- 0 +	+ +	+ +	+ + +	+ + + +	+ + +	+ + +
f(x)	I$_{+\infty}$	1	0	0.17	½	1	2	5.82	$_{+\infty}$I