Calculul ecuatiilor matriciale

Fie A, B$\in M_{m\times m}(C)$, A = $\begin{vmatrix} a_{11} & a_{12} & a_{13} \ldots\ldots a_{1m} \\ a_{21} & a_{22} & a_{23} \ldots\ldots a_{2m} \\ a_{31} & a_{32} & a_{33} \ldots\ldots a_{3m} \\ \ldots & \ldots & \ldots \\ a_{m1} & a_{m2} & a_{m3} \ldots\ldots a_{mm} \end{vmatrix}$, B= $\begin{vmatrix} b_{11} & b_{12} & b_{13} \ldots\ldots b_{1m} \\ b_{21} & b_{22} & b_{23} \ldots\ldots b_{2m} \\ b_{31} & b_{32} & b_{33} \ldots\ldots b_{3m} \\ \ldots & \ldots & \ldots \\ b_{m1} & b_{m2} & b_{m3} \ldots\ldots b_{mm} \end{vmatrix}$

astfel încât A *så fie ne singularå (deci så existe*A^{-1}*). Så considerâm ecuatiile matriceale :*

$$AX=B,\ YA=B$$

înmultind prima ecuatie la stânga cu A^{-1} *si pe a doua la dreapta cu* A^{-1}*, se obtine:*

$$A^{-1}(AX)= A^{-1}B,\ (YA)A^{-1}= BA^{-1}$$

folosind asociativitatea matricilor se obtine:

$$(A^{-1}A)X= A^{-1}B,\ Y(AA^{-1})= BA^{-1}$$

dar $(A^{-1}A)=I_m$ *si folosind proprietatea matricii identice,se va obtine:*

$$X= A^{-1}B,\ Y= BA^{-1}$$

iar prin calculul $A^{-1}B$ *si* BA^{-1} *se va obtine* X, Y. *De obicei* X,Y *sunt diferite deoarece înmultirea matricilor nu este comutativå:*

Så se rezolve urmåtoarele ecuatii matriciale:

1. $\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} \cdot X = \begin{vmatrix} 5 & 6 \\ 6 & 8 \end{vmatrix}$,notez A= $\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix}$,B= $\begin{vmatrix} 5 & 6 \\ 6 & 8 \end{vmatrix}$

$A^{-1}\cdot/A\cdot X=B \Rightarrow A^{-1}\cdot(A\cdot X)= A^{-1}B\Rightarrow (A^{-1}\cdot A)\cdot X= A^{-1}B\Rightarrow I_2\cdot X= A^{-1}B\Rightarrow X= A^{-1}B$

$\det A = 2\cdot3 – 2 = 4$

$A^{-1}=(1/\det A)\cdot A^*$

$A^*=\begin{vmatrix} a^*_{11} & a^*_{21} \\ a^*_{12} & a^*_{22} \end{vmatrix}$

$a^*_{11} = (-1)^2 \cdot 3 = 3$
$a^*_{12} = (-1)^3 \cdot 2 = -2$
$a^*_{21} = (-1)^3 \cdot 1 = -1$
$a^*_{22} = (-1)^4 \cdot 2 = 2$

$$A^* = \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix}$$

$$A^{-1} = (1/4) \cdot \begin{vmatrix} 3 & -1 \\ -2 & 2 \end{vmatrix} = \begin{vmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{vmatrix}$$

$$X = \begin{vmatrix} 3/4 & -1/4 \\ -1/2 & 1/2 \end{vmatrix} \cdot \begin{vmatrix} 5 & 6 \\ 6 & 8 \end{vmatrix} = \begin{vmatrix} 9/4 & 5/2 \\ 1/2 & 1 \end{vmatrix}$$

2. $X \cdot \begin{vmatrix} -1 & -2 \\ 5 & 8 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -3 & 6 \end{vmatrix}$, notez $A = \begin{vmatrix} -1 & -2 \\ 5 & 8 \end{vmatrix}$, $B = \begin{vmatrix} 1 & 2 \\ -3 & 6 \end{vmatrix}$

$X \cdot A = B / \cdot A^{-1} \Rightarrow (X \cdot A) \cdot A^{-1} = B \cdot A^{-1} \Rightarrow X \cdot (A \cdot A^{-1}) = B \cdot A^{-1} \Rightarrow X \cdot I_2 = B \cdot A^{-1} \Rightarrow X = B \cdot A^{-1}$

$\det A = -8 + 10 = 2$
$A^{-1} = (1/\det A) \cdot A^*$

$$A^* = \begin{vmatrix} a^*_{11} & a^*_{21} \\ a^*_{12} & a^*_{22} \end{vmatrix}$$

$a^*_{11} = (-1)^2 \cdot 8 = 8$
$a^*_{12} = (-1)^3 \cdot (-2) = 2$
$a^*_{21} = (-1)^3 \cdot 5 = -5$
$a^*_{22} = (-1)^4 \cdot (-1) = -1$

$$A^* = \begin{vmatrix} 8 & 2 \\ -5 & -1 \end{vmatrix}$$

$$\begin{vmatrix} 8 & 2 \\ & \end{vmatrix} \quad \begin{vmatrix} 4 & 1 \\ & \end{vmatrix}$$

$A^{-1}=(1/2)\cdot \qquad =$

$\qquad -5 \quad -1 \qquad -5/2 \; -1/2$

$$X=\begin{vmatrix} 7 & 2 \\ -3 & 5 \end{vmatrix} \cdot \begin{vmatrix} 4 & 1 \\ -5/2 & -1/2 \end{vmatrix} = \begin{vmatrix} 23 & 6 \\ -49/4 & -11/2 \end{vmatrix}$$

3. $\begin{vmatrix} -1 & 2 \\ -3 & 8 \end{vmatrix} \cdot X \cdot \begin{vmatrix} 4 & 6 \\ 5 & 8 \end{vmatrix} = \begin{vmatrix} 8 & 13 \\ -4 & -7 \end{vmatrix}$,notez $A=\begin{vmatrix} -1 & 2 \\ -3 & 8 \end{vmatrix}$,$B=\begin{vmatrix} 4 & 6 \\ 5 & 8 \end{vmatrix}$,$C=\begin{vmatrix} 8 & 13 \\ -4 & -7 \end{vmatrix}$

$A\cdot X\cdot B=C /\cdot B^{-1} \Rightarrow (A\cdot X\cdot B)\cdot B^{-1}=C\cdot B^{-1} \Rightarrow (A\cdot X)\cdot(B\cdot B^{-1})=C\cdot B^{-1} \Rightarrow (A\cdot X)\cdot I_2=C\cdot B^{-1} \Rightarrow (A^{-1})\cdot/A\cdot X=C\cdot B^{-1} \Rightarrow A^{-1}(A\cdot X)=A^{-1}\cdot C\cdot B^{-1} \Rightarrow (A^{-1}\cdot A)\cdot X=A^{-1}\cdot C\cdot B^{-1} \Rightarrow I_2\cdot X=A^{1}\cdot C\cdot B^{-1} \Rightarrow X=A^{-1}\cdot C\cdot B^{-1}$

$\det A = -8 +6 =-2$

$A^{-1}=(1/ \det A)\cdot A^*$

$$A^*=\begin{vmatrix} a^*_{11} & a^*_{21} \\ a^*_{12} & a^*_{22} \end{vmatrix}$$

$a^*_{11} =(-1)^2 \cdot 8= 8$

$a^*_{12} =(-1)^3 \cdot(-3)=3$

$a^*_{21} =(-1)^3 \cdot 2= -2$

$a^*_{22} =(-1)^4 \cdot(-1)= -1$

$$A^*=\begin{vmatrix} 8 & -2 \\ 3 & -1 \end{vmatrix}$$

$$A^{-1}=[1/-(2)] \cdot \begin{vmatrix} 8 & -2 \\ 3 & -1 \end{vmatrix} = \begin{vmatrix} -4 & 1 \\ -3/2 & 1/2 \end{vmatrix}$$

$\det B= 32-30 =2$

$B^{-1}=(1/ \det B)\cdot B^*$

$$B^*=\begin{vmatrix} b^*_{11} & b^*_{21} \\ b^*_{12} & b^*_{22} \end{vmatrix}$$

$b^*_{11} = (-1)^2 \cdot 8 = 8$
$b^*_{12} = (-1)^3 \cdot 5 = -5$
$b^*_{21} = (-1)^3 \cdot 6 = -6$
$b^*_{22} = (-1)^4 \cdot 4 = 4$

$$B^* = \begin{vmatrix} 8 & -6 \\ -5 & 4 \end{vmatrix}$$

$$B^{-1} = (1/2) \cdot \begin{vmatrix} 8 & -6 \\ -5 & 4 \end{vmatrix} = \begin{vmatrix} 4 & -3 \\ -5/2 & 2 \end{vmatrix}$$

$$X = \begin{vmatrix} -4 & 1 \\ -3/2 & 1/2 \end{vmatrix} \cdot \begin{vmatrix} 8 & 13 \\ -4 & -7 \end{vmatrix} \cdot \begin{vmatrix} 4 & -3 \\ -5/2 & 2 \end{vmatrix} = \begin{vmatrix} -36 & -59 \\ -14 & -23 \end{vmatrix} \cdot \begin{vmatrix} 4 & -3 \\ -5/2 & 2 \end{vmatrix} = \begin{vmatrix} 7/2 & -10 \\ 3/2 & -4 \end{vmatrix}$$

4. $X \cdot \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} -1 & 5 & 3 \\ 2 & 1 & -1 \\ -3 & 4 & -5 \end{vmatrix}$,notez $A = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 2 & 1 \end{vmatrix}$,$B = \begin{vmatrix} -1 & 5 & 3 \\ 2 & 1 & -1 \\ -3 & 4 & -5 \end{vmatrix}$

$X \cdot A = B / \cdot A^{-1} \Rightarrow (X \cdot A) \cdot A^{-1} = B \cdot A^{-1} \Rightarrow X \cdot (A \cdot A^{-1}) = B \cdot A^{-1} \Rightarrow X \cdot I_2 = B \cdot A^{-1} \Rightarrow X = B \cdot A^{-1}$

$\det A = 1 - 4 + 3 - 4 = -4$
$A^{-1} = (1/\det A) \cdot A^*$

$$A^* = \begin{vmatrix} a^*_{11} & a^*_{21} & a^*_{31} \\ a^*_{12} & a^*_{22} & a^*_{32} \\ a^*_{13} & a^*_{23} & a^*_{33} \end{vmatrix}$$

$a^*_{11} = (-1)^2 \cdot (-3) = -3$
$a^*_{12} = (-1)^3 \cdot 2 = -2$
$a^*_{13} = (-1)^4 \cdot 1 = 1$
$a^*_{21} = (-1)^3 \cdot (-4) = 4$
$a^*_{22} = (-1)^4 \cdot 4 = 4$
$a^*_{23} = (-1)^5 \cdot 4 = 4$
$a^*_{31} = (-1)^4 \cdot 1 = 1$

$a^*_{32} = (-1)^5 \cdot 2 = -2$
$a^*_{33} = (-1)^6 \cdot 1 = 1$

$$A^* = \begin{vmatrix} -3 & 4 & 1 \\ -2 & 4 & -2 \\ 1 & -4 & 1 \end{vmatrix}$$

$$A^{-1} = [1/(-4)] \cdot \begin{vmatrix} -3 & 4 & 1 \\ -2 & 4 & -2 \\ 1 & -4 & 1 \end{vmatrix}$$

$$X = [1/(-4)] \cdot \begin{vmatrix} -1 & 5 & 3 \\ 2 & 1 & -1 \\ -3 & 4 & -5 \end{vmatrix} \cdot \begin{vmatrix} -3 & 4 & 1 \\ -2 & 4 & -2 \\ 1 & -4 & 1 \end{vmatrix} = [1/(-4)] \cdot \begin{vmatrix} -4 & 4 & -8 \\ -9 & 16 & -1 \\ -4 & 24 & -16 \end{vmatrix} = \begin{vmatrix} 1 & -1 & 2 \\ 9/4 & -4 & 1/4 \\ 1 & -6 & 4 \end{vmatrix}$$

5. $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot X = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$,notez $A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$,$B = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

$A^{-1} \cdot / A \cdot X = B \Rightarrow A^{-1} \cdot (A \cdot X) = A^{-1}B \Rightarrow (A^{-1} \cdot A) \cdot X = A^{-1}B \Rightarrow I_2 \cdot X = A^{-1}B \Rightarrow X = A^{-1}B$

$$\det A = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1$$

$A^{-1} = (1/\det A) \cdot A^*$

$$A^* = \begin{vmatrix} a^*_{11} & a^*_{21} & a^*_{31} & a^*_{41} \\ a^*_{12} & a^*_{22} & a^*_{32} & a^*_{42} \\ a^*_{13} & a^*_{23} & a^*_{33} & a^*_{43} \\ a^*_{14} & a^*_{24} & a^*_{34} & a^*_{44} \end{vmatrix}$$

$a^*_{11} = (-1)^2 \cdot 1 = 1$ $\quad a^*_{21} = (-1)^3 \cdot 1 = -1$ $\quad a^*_{31} = (-1)^4 \cdot 0 = 0$ $\quad a^*_{41} = (-1)^5 \cdot 0 = 0$
$a^*_{12} = (-1)^3 \cdot 0 = 0$ $\quad a^*_{22} = (-1)^4 \cdot 1 = 1$ $\quad a^*_{32} = (-1)^5 \cdot 1 = -1$ $\quad a^*_{42} = (-1)^6 \cdot 0 = 0$
$a^*_{13} = (-1)^4 \cdot 0 = 0$ $\quad a^*_{23} = (-1)^5 \cdot 0 = 0$ $\quad a^*_{33} = (-1)^6 \cdot 1 = 1$ $\quad a^*_{43} = (-1)^7 \cdot 1 = -1$
$a^*_{14} = (-1)^5 \cdot 0 = 0$ $\quad a^*_{24} = (-1)^6 \cdot 0 = 0$ $\quad a^*_{34} = (-1)^7 \cdot 0 = 0$ $\quad a^*_{44} = (-1)^8 \cdot 1 = 1$

$$\begin{vmatrix} 1 & -1 & 0 & 0 \\ & & & \\ & & & \\ & & & \end{vmatrix}$$

$A^*= \begin{matrix} 0 & 1 & -1 & 0 \end{matrix} = A^{-1}$

$\begin{matrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{matrix}$

$$X = \begin{vmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

6. $\begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{vmatrix} \cdot X \cdot \begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$,notez $A=\begin{vmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ -1 & 2 & 1 \end{vmatrix}$,$B=\begin{vmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{vmatrix}$,$C=\begin{vmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$

$A\cdot X\cdot B=C/\cdot B^{-1}\Rightarrow(A\cdot X\cdot B)\cdot B^{-1}=C\cdot B^{-1}\Rightarrow(A\cdot X)\cdot(B\cdot B^{-1})=C\cdot B^{-1}\Rightarrow(A\cdot X)\cdot I_2=C\cdot B^{-1}\Rightarrow$ $(A^{-1})\cdot/A\cdot X=C\cdot B^{-1}\Rightarrow A^{-1}(A\cdot X)=A^{-1}\cdot C\cdot B^{-1}\Rightarrow(A^{-1}\cdot A)\cdot X=A^{-1}\cdot C\cdot B^{-1}\Rightarrow I_2\cdot X=A^{1}\cdot C\cdot B^{-1}\Rightarrow X=A^{-1}\cdot C\cdot B^{-1}$

$\det A = -2+6-3-2=-1$

$A^{-1}=(1/\det A)\cdot A^*$

$$A^*=\begin{vmatrix} a^*_{11} & a^*_{21} & a^*_{31} \\ a^*_{12} & a^*_{22} & a^*_{32} \\ a^*_{13} & a^*_{23} & a^*_{33} \end{vmatrix}$$

$a^*_{11} =(-1)^2 \cdot(-1)= -1$ $a^*_{21} =(-1)^3 \cdot(-4)= 4$ $a^*_{31} =(-1)^4 \cdot 3= 3$

$a^*_{12} =(-1)^3 \cdot 1=-1$ $a^*_{22} =(-1)^4 \cdot 5= 5$ $a^*_{32} =(-1)^5 \cdot(-3)= 3$

$a^*_{13} =(-1)^4 \cdot 1= 1$ $a^*_{23} =(-1)^5 \cdot 6= -6$ $a^*_{33} =(-1)^6 \cdot(-4)= -4$

$$A^*=\begin{vmatrix} -1 & 4 & 3 \\ -1 & 5 & 3 \\ 1 & -6 & -4 \end{vmatrix}$$

$$A^{-1}=[1/(-1)] \cdot\begin{vmatrix} -1 & 4 & 3 \\ -1 & 5 & 3 \\ 1 & -6 & -4 \end{vmatrix} = \begin{vmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{vmatrix}$$

$\det B= 1=1$

$B^{-1}=(1/\det B)\cdot B^*$

$$B^*=\begin{vmatrix} b^*_{11} & b^*_{21} & b^*_{31} \\ b^*_{12} & b^*_{22} & b^*_{32} \\ b^*_{13} & b^*_{23} & b^*_{33} \end{vmatrix}$$

$b^*_{11} = (-1)^2 \cdot 1 = 1$ $\quad b^*_{21} = (-1)^3 \cdot 2 = -2$ $\quad b^*_{31} = (-1)^4 \cdot 7 = 7$
$b^*_{12} = (-1)^3 \cdot 0 = 0$ $\quad b^*_{22} = (-1)^4 \cdot 1 = 1$ $\quad b^*_{32} = (-1)^5 \cdot 2 = -2$
$b^*_{13} = (-1)^4 \cdot 0 = 0$ $\quad b^*_{23} = (-1)^5 \cdot 0 = 0$ $\quad b^*_{33} = (-1)^6 \cdot 1 = 1$

$$B^* = \begin{vmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix} = B^{-1}$$

$$X = \begin{vmatrix} 1 & -4 & -3 \\ 1 & -5 & -3 \\ -1 & 6 & 4 \end{vmatrix} \cdot \begin{vmatrix} 0 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -5 & -8 \\ 0 & -6 & -9 \\ 0 & 7 & 11 \end{vmatrix} \cdot \begin{vmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -5 & 2 \\ 0 & -6 & 3 \\ 0 & 7 & -3 \end{vmatrix}$$

7. $X \cdot \begin{vmatrix} 5 & 3 & 4 \\ -6 & -3 & -5 \\ 4 & 2 & 2 \end{vmatrix} = (3\ \ 2\ \ 1)$,notez $A = \begin{vmatrix} 5 & 3 & 4 \\ -6 & -3 & -5 \\ 4 & 2 & 2 \end{vmatrix}$,$B = (\ 3\ 2\ 1)$

$X \cdot A = B / \cdot A^{-1} \Rightarrow (X \cdot A) \cdot A^{-1} = B \cdot A^{-1} \Rightarrow X \cdot (A \cdot A^{-1}) = B \cdot A^{-1} \Rightarrow X \cdot I_2 = B \cdot A^{-1} \Rightarrow X = B \cdot A^{-1}$

$\det A = -30-48-60+48+50+36 = -4$

$A^{-1} = (1/ \det A) \cdot A^*$

$$A^* = \begin{vmatrix} a^*_{11} & a^*_{21} & a^*_{31} \\ a^*_{12} & a^*_{22} & a^*_{32} \\ a^*_{13} & a^*_{23} & a^*_{33} \end{vmatrix}$$

$a^*_{11} = (-1)^2 \cdot 4 = 4$ $\quad a^*_{21} = (-1)^3 \cdot (-2) = 2$ $\quad a^*_{31} = (-1)^4 \cdot (-3) = -3$
$a^*_{12} = (-1)^3 \cdot 8 = -8$ $\quad a^*_{22} = (-1)^4 \cdot (-6) = -6$ $\quad a^*_{32} = (-1)^5 \cdot (-1) = 1$
$a^*_{13} = (-1)^4 \cdot 0 = 0$ $\quad a^*_{23} = (-1)^5 \cdot (-2) = 2$ $\quad a^*_{33} = (-1)^6 \cdot 3 = 3$

$$A^* = \begin{vmatrix} 4 & 2 & -3 \\ -8 & -6 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

$$X = [1/(-4)] \cdot (3\ \ 2\ \ 1) \cdot \begin{vmatrix} 4 & 2 & -3 \\ -8 & -6 & 1 \\ 0 & 2 & 3 \end{vmatrix} = [1/(-4)] \cdot (-4\ -4\ -4) = (\ 1\ \ 1\ \ 1)$$

8. $X \cdot \begin{vmatrix} 2 & 0 & 1 \\ 4 & s & 2 \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{vmatrix}$ $s \in R$,notez $A = \begin{vmatrix} 2 & 0 & 1 \\ 4 & s & 2 \\ 1 & 1 & 1 \end{vmatrix}$,$B = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{vmatrix}$

$X\cdot A=B/\cdot A^{-1}\Rightarrow(X\cdot A)\cdot A^{-1}=B\cdot A^{-1}\Rightarrow X\cdot(A\cdot A^{-1})=B\cdot A^{-1}\Rightarrow X\cdot I_2=B\cdot A^{-1}\Rightarrow X=B\cdot A^{-1}$

$A^{-1}=(1/\det A)\cdot A^*$

$\det A = 2s+4-s-4=s$

Cazul I $s=0 \Rightarrow \nexists\ A^{-1}$

Cazul II $s\neq 0\Rightarrow \exists\ A^{-1}\Rightarrow \det A=s$

$A^{-1}=(1/\det A)\cdot A^*$

$$A^*=\begin{vmatrix} a^*_{11} & a^*_{21} & a^*_{31} \\ a^*_{12} & a^*_{22} & a^*_{32} \\ a^*_{13} & a^*_{23} & a^*_{33} \end{vmatrix}$$

$a^*_{11}=(-1)^2\cdot(s-2)= s-2$

$a^*_{12}=(-1)^3\cdot 2=-2$

$a^*_{13}=(-1)^4\cdot(4-s)=4-s$

$a^*_{21}=(-1)^3\cdot(-1)=1$

$a^*_{22}=(-1)^4\cdot 1= 1$

$a^*_{23}=(-1)^5\cdot 2=-2$

$a^*_{31}=(-1)^4\cdot(-s)=-s$

$a^*_{32}=(-1)^5\cdot 0=0$

$a^*_{33}=(-1)^6\cdot 2s=2s$

$$A^*=\begin{vmatrix} s-2 & 1 & -s \\ -2 & 1 & 0 \\ 4-s & -2 & 2s \end{vmatrix}$$

$$X = 1/s\cdot\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{vmatrix}\cdot\begin{vmatrix} s-2 & 1 & -s \\ -2 & 1 & 0 \\ 4-s & -2 & 2s \end{vmatrix}=1/s\cdot\begin{vmatrix} 6-2s & -3 & 5s \\ 2-2s & -3 & 4s \\ 8-3s & -4 & 5s \end{vmatrix}=\begin{vmatrix} (6-2s)/s & -3/s & 5 \\ (2-2s)/s & -3/s & 4 \\ (8-3s)/s & -4/s & 5 \end{vmatrix}$$

9. $$X\cdot\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix}=\begin{pmatrix} 6 & 9 & 8 \\ & & \\ 0 & 1 & 6 \end{pmatrix}, \text{notez } A=\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{pmatrix}, B=\begin{pmatrix} 6 & 9 & 8 \\ & & \\ 0 & 1 & 6 \end{pmatrix}$$

$X\cdot A=B/\cdot A^{-1}\Rightarrow(X\cdot A)\cdot A^{-1}=B\cdot A^{-1}\Rightarrow X\cdot(A\cdot A^{-1})=B\cdot A^{-1}\Rightarrow X\cdot I_2=B\cdot A^{-1}\Rightarrow X=B\cdot A^{-1}$

$\det A = 3+24+24-27-16-4 = 4$

$A^{-1}=(1/\det A)\cdot A^*$

$$\begin{pmatrix} a^*_{11} & a^*_{21} & a^*_{31} \\ & & \\ & & \end{pmatrix}$$

$A^* = \begin{matrix} a^*_{12} & a^*_{22} & a^*_{32} \\ a^*_{13} & a^*_{23} & a^*_{33} \end{matrix}$

$a^*_{11} = (-1)^2 \cdot (-13) = -13$
$a^*_{12} = (-1)^3 \cdot (-10) = 10$
$a^*_{13} = (-1)^4 \cdot (-1) = -1$
$a^*_{21} = (-1)^3 \cdot (-10) = 10$
$a^*_{22} = (-1)^4 \cdot (-8) = -8$
$a^*_{23} = (-1)^5 \cdot (-2) = 2$
$a^*_{31} = (-1)^4 \cdot (-1) = -1$
$a^*_{32} = (-1)^5 \cdot (-2) = 2$
$a^*_{33} = (-1)^6 \cdot (-1) = -1$

$$A^* = \begin{pmatrix} -13 & 10 & -1 \\ 10 & -8 & 2 \\ -1 & 2 & -1 \end{pmatrix}$$

$$X = (1/4) \cdot \begin{pmatrix} 6 & 9 & 8 \\ 0 & 1 & 6 \end{pmatrix} \cdot \begin{pmatrix} -13 & 10 & -1 \\ 10 & -8 & 2 \\ -1 & 2 & -1 \end{pmatrix} = (1/-4) \cdot \begin{pmatrix} 4 & 4 & 4 \\ 4 & 4 & -4 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$