Trasarea graficului unei functii

In studiul variatiei unei functii si trasarea graficului se parcurg urmatoarele etape de determinare succesiva a unor elemente caracteristice ale functiei:

I. *Domeniul de definitie:*

a) Determinarea domeniului de definitie (in cazul expresiilor rationale numitorul trebuie sa fie diferit de zero; in cazul celor irationale cantitatea de sub radical trebuie sa fie cel putin zero)

b) Intersectia graficului cu axa *Ox*: *f(x)=0*

c) Intersectia graficului cu axa *Oy:f(0)=*…

d) Calculul limitelor:

$$\lim_{x->\infty} f(x) = ... \qquad \text{si} \qquad \lim_{x->-\infty} f(x) = ...$$

II. *Semnul functiei:*

a) Determinarea paritatii sau imparitatii functiei(daca functia este para,*f(x)=f(-x),*atunci graficul este simetric fata de axa ordonatelor; daca functia este impara,*-f(x)=f(-x)*, atunci graficul este simetric fata de originea axelor).

b) Determinarea periodicitatii functiei si, in cazul functiilor periodice, a perioadei *T*.

c) Continuitatea functiei.

III. *Asimptote:*

a) orizontale;

b) oblice;

c) verticale.

IV. *Studiul primei derivate:*

a) Se determina multimea E` inclusa in domeniul de definitie, pe care functia *f* este derivabila si apoi se calculeaza *f `(x).*

b) Se rezolva ecuatia *f `(x)=0*, ale carei radacini sunt, eventual, puncte critice ale functiei.

c) Se calculeaza valoarile functiei pe radacinile derivatei I.

d) Determinarea semnului derivatei I, care da monotonia functiei.

V. *Studiul derivatei a doua:*

a) Se determina multimea E`` inclusa in E`, pe care functia *f `* este derivabila si apoi se calculeaza *f ``(x).*

b) Se rezolva ecuatia *f ``(x)=0,* iar radacinile pot fi puncte de inflexiune.

c) Se calculeaza valoarile functiei pe radacinile derivatei II.

d) Determinarea semnului derivateiei II, care ne da convexitatea sau concavitatea functiei.

VI. *Formarea tabloului de variatie a functiei f* – tablou in care se trec pentru sistematizare, rezultateleobtinute la punctele precedente:

x	
f `(x)	
f ``(x)	
f(x)	

VII. *Trasarea graficului functiei:*- conform rezultatelorsistematizate in tabloul de variatie – intr-un sistem de axe carteziene.

APLICATII:

1. Sa se studieze variatia functiilor si sa se reprezinte grafic:

a) $f(x)=\sqrt{|x^2-1|}-x$

I. a) $D=(-\infty,+\infty);$

$$f(x)=\begin{cases}\sqrt{x^2-1}-x, & \text{daca } x\in(-\infty,-1)\cup(1,\infty)\\ \sqrt{1-x^2}-x, & \text{daca } x\in[-1,1]\end{cases}$$

b) $G_f\cap Ox:\ f(x)=0\Rightarrow\sqrt{|x^2-1|}-x=0$

$$\Leftrightarrow |x^2-1|=x^2\Leftrightarrow x^2-1=\pm x^2\Rightarrow 2x^2=1\Rightarrow x=\frac{\sqrt{2}}{2}\quad\Rightarrow\ A(\frac{\sqrt{2}}{2},0)$$

c) $G_f\cap Oy:\ f(0)=1\ \Rightarrow\ B(0,1);$

d) $\lim\limits_{n\to-\infty}f(x)=\lim\limits_{n\to-\infty}\sqrt{x^2-1}-x=+\infty$

$$\lim_{n\to\infty}f(x)=\lim_{n\to\infty}\sqrt{x^2-1}-x=\lim_{n\to\infty}\frac{x^2-1-x^2}{\sqrt{x^2-1}+x}=0$$

II. $f(-x)=\sqrt{x^2-1}+x$

III. asimptote orizontale : - spre $-\infty$ ____

- spre $+\infty$ $\quad y = 0$

asimptota oblica spre $-\infty$: y = mx + n

$$m = \lim_{x\to-\infty}\frac{f(x)}{x} = \lim_{x\to-\infty}\frac{\sqrt{x^2-1}-x}{x} = \lim_{x\to-\infty}\frac{-x\sqrt{1-\frac{1}{x^2}}-x}{x} = \lim_{x\to-\infty}\frac{-x\left(\sqrt{1-\frac{1}{x^2}}+1\right)}{x} = -2$$

$$n = \lim_{x\to-\infty}\left(f(x)-mx\right) = \lim_{x\to-\infty}\left(\sqrt{x^2-1}-x+2x\right) = \lim_{x\to-\infty}\left(\sqrt{x^2-1}+x\right) = 0$$

$\Rightarrow y = -2x$ este asimptota oblica spre $-\infty$

asimptote verticale : ___

$$\text{IV. } f`(x) = \begin{cases} \dfrac{x}{\sqrt{x^2-1}} - 1 \ , \ x \in (-\infty,-1)\cup(1,\infty) \\ -\dfrac{x}{\sqrt{1-x^2}} - 1 \ , \ x \in (-1,1) \end{cases}$$

$$\left.\begin{array}{l} f`_s(-1) = \lim\limits_{\substack{x\to-1\\ x<-1}} f`(x) = \lim\limits_{\substack{x\to-1\\ x<-1}} \dfrac{x}{\sqrt{x^2-1}} - 1 = \dfrac{-1}{0_+} - 1 = -\infty \\ f`_d(-1) = \lim\limits_{\substack{x\to-1\\ x>-1}} f`(x) = \lim\limits_{\substack{x\to-1\\ x>-1}} \dfrac{x}{\sqrt{1-x^2}} - 1 = \dfrac{-1}{0_-} = \infty \\ f(-1) = 1 \end{array}\right\} \Rightarrow M_1(-1,+1) \text{ - punct de intoarcere;}$$

$$\left.\begin{array}{l} f`_s(1) = \lim\limits_{\substack{x\to1\\ x<1}} f`(x) = \lim\limits_{\substack{x\to1\\ x<1}} \dfrac{x}{\sqrt{1-x^2}} - 1 = \dfrac{1}{0_-} - 1 = -\infty \\ f`_d(1) = \lim\limits_{\substack{x\to1\\ x>1}} f`(x) = \lim\limits_{\substack{x\to1\\ x>1}} \dfrac{x}{\sqrt{x^2-1}} - 1 = \dfrac{1}{0_+} = \infty \\ f(1) = -1 \end{array}\right\} \Rightarrow M_2(1,-1) \text{ - punct de intoarcere;}$$

$$f`(x) = 0 \Rightarrow -\frac{x}{\sqrt{1-x^2}} - 1 = 0 \Rightarrow x = -\frac{\sqrt{2}}{2}$$

$$f\left(-\frac{\sqrt{2}}{2}\right) = \sqrt{2} \Rightarrow C\left(-\frac{\sqrt{2}}{2}, \sqrt{2}\right)$$

x	-∞ -1 0 1 +∞
$f`(x)$	- - - -∞ | +∞ + 0 - - - - - - -∞ | +∞ + +
$f(x)$	+∞ 1 1 0 -1 0

➔ in –1 si 1 avem puncte de intoarcere.

b) $f(x)=\dfrac{|x|-3}{1+|x|}$

I. a) $D=(-\infty,\infty)$;

$$f(x)=\begin{cases}\dfrac{x-3}{x+1}, & x\in[o,\infty)\\ \dfrac{x+3}{x-1}, & x\in(-\infty,0)\end{cases}$$

b) $G_f\cap Ox:\quad f(x)=0\Rightarrow\dfrac{x-3}{x+1}=0\Rightarrow x-3=0\Rightarrow x=3\qquad A(3,0)$

$\Rightarrow\dfrac{x+3}{x-1}=0\Rightarrow x=-3\qquad A`(-3,0)$

c) $G_f\cap Ox:\quad f(0)=-3\quad\Rightarrow\quad B(0,-3)$

d) $\lim\limits_{x\to\pm\infty}f(x)=1$

II. $f(-x)=\dfrac{|x|-3}{1+|x|}=f(x)\Rightarrow$ functia este para, deci are graficul simetric fata de axa Oy

$\Rightarrow$ este suficient sa studiem functia pe restrictia sa $[0,\infty)$

III. asimptota orizontala: $y=1$

asimptota oblica: ___

asimptota verticala: _____

IV. $f`(x)=\left(\dfrac{x-3}{x+1}\right)^{`}=\dfrac{x+1-x+3}{(x+1)^2}=\dfrac{4}{(x+1)^2}\Rightarrow f`(x)>0\Rightarrow f$ este strict crescatoare pe $[0,\infty)$

$\Rightarrow B(0.-3)$ este punct unghiular al functiei pe R (datorita simetriei sale fata de axa Oy)

V. $f``(x)=(f`(x))`=\left(\dfrac{4}{(x+1)^2}\right)^{`}=-\dfrac{8}{(x+1)^4}\Rightarrow f``(x)<0\Rightarrow$ f este concava pe $(0,\infty)$

VI. Tabloul de variatie:

x	0			3						+∞
f `(x)	+	+	+	+	+	+	+	+	+	+
f``(x)	-	-	-	-	-	-	-	-	-	-
f(x)	-3			0						1

2. Se considera functia:

$$f : D \to R \qquad f(x) = \frac{2x^2+1}{x(x+k)}$$

unde D este domeniul maxim de definitie iar k partine lui R. Sa se traseze graficul functiei f stiind ca trce prin punctul (1,1).

<u>Demonstratie:</u>

Intrucat $M(1,1) \in G_f \Rightarrow f(1) = 1 \Leftrightarrow \frac{3}{1+k} = 1 \Rightarrow k = 2;$

$$\Rightarrow f(x) = \frac{2x^2+1}{x(x+2)}$$

$\Rightarrow D = R - \{-2,0\}$

I. a) $f : R - \{-2,0\} \to R$

b) $G_f \cap Ox : f(x) = 0 \Rightarrow 2x^2 + 1 = 0$

$\Delta < 0 \Rightarrow G_f$ nu intersecteaza axa Ox

c) $G_f \cap Oy : f(0) =$ _____

d) $\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x^2+1}{x(x+2)} = 2$

II. $f(-x) = \frac{2x^2+1}{(-x)(-x+2)} = \frac{2x^2+1}{x(x-2)} \Rightarrow f$ este oarecare

III. asimptote orizontale : $y = 2$ spre $\pm\infty$

asimptote oblice : _____

asimptote verticale :

$$\left.\begin{aligned} \lim_{\substack{x\to-2\\ x<-2}} f(x) = \lim_{\substack{x\to-2\\ x<-2}} \frac{2x^2+1}{x(x+2)} = \frac{9}{0_+} = \infty \\ \lim_{\substack{x\to-2\\ x>-2}} f(x) = \lim_{\substack{x\to-2\\ x>-2}} \frac{2x^2+1}{x(x+2)} = \frac{9}{0\ -} = -\infty \end{aligned}\right\} \Rightarrow x = -2 \text{ asimptota verticala}$$

$$\left.\begin{array}{l}\lim\limits_{\substack{x\to 0\\ x<0}} f(x)=\lim\limits_{\substack{x\to 0\\ x<0}}\dfrac{2x^2+1}{x(x+2)}=\dfrac{1}{0_-}=-\infty\\[2ex] \lim\limits_{\substack{x\to 0\\ x>0}} f(x)=\lim\limits_{\substack{x\to 0\\ x>0}}\dfrac{2x^2+1}{x(x+2)}=\dfrac{1}{0_+}=+\infty\end{array}\right\}\Rightarrow x=0 \text{ este asimptota verticala}$$

IV. $f`(x)=\left(\dfrac{2x^2+1}{x(x+2)}\right)`=\dfrac{4x^2-2x-2}{x^2(x+2)^2}$

$$f`(x)=0\Rightarrow 4x^2-2x-2=0\Rightarrow x_1=1;\ \ x_2=-\frac{1}{2}$$

$$f(1)=1;\quad f(-\frac{1}{2})=-2$$

V.

x	-∞	-2	-1/2	0	1	∞
f `(x)	+ + +	+ + +	0 - - -	- - - -	0 + + +	
f(x)	2 +∞	-∞	-2 -∞	+∞	1	

3. Sa se reprezinte grafic functia:

$$f(x)=\frac{2(x-1)^2}{\sqrt{4x^2+2x+1}}$$

Demonstratie :

I. a) $f:R\to R$

b) $G_f\cap Ox: f(x)=0\Rightarrow 2(x-1)^2=0\Rightarrow x=1\quad O(1,0)$

c) $G_f\cap Oy: f(0)=2\quad A(0,2)$

d) $\lim\limits_{x\to\pm\infty} f(x)=\lim\limits_{x\to\pm\infty}\frac{2(x-1)^2}{\sqrt{4x^2+2x+1}}=+\infty$

II. $f(-x)=\frac{2(-x-1)^2}{\sqrt{4x^2-2x+1}}\Rightarrow$ functia este oarecare

III. asimptote orizontale : ______

asimptote oblice : $y=mx+n$

$$m_1=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{2(x-1)^2}{x\sqrt{4x^2+2x+1}}=1$$

$$n_1=\lim_{x\to\infty}[f(x)-x]=\lim_{x\to\infty}\frac{2(x-1)^2-x\sqrt{4x^2+2x+1}}{\sqrt{4x^2+2x+1}}=$$

$$=\lim_{x\to\infty}\frac{4(x-1)^4-x^2(4x^2+2x+1)}{\sqrt{4x^2+2x+1}\left[2(x-1)^2+\sqrt{4x^2+2x+1}\right]}=$$

$$=\lim_{x\to\infty}\frac{-18x^3+23x^2-16x+4}{x^3\sqrt{4+2\frac{1}{x}+\frac{1}{x^2}}*\left[2\left(1-\frac{1}{x}\right)^2+\sqrt{4+2\frac{1}{x}+\frac{1}{x^2}}\right]}=-\frac{9}{4}$$

$\Rightarrow y=x-\frac{9}{4}$ este asimptota oblica pentru $x\to+\infty$

$$m_2=\lim_{x\to-\infty}\frac{f(x)}{x}=\lim_{x\to-\infty}\frac{2(x-1)^2}{x\sqrt{4x^2+2x+1}}=-1$$

$$n_2=\lim_{x\to-\infty}[f(x)+x]=\lim_{x\to-\infty}\frac{2(x-1)^2+x\sqrt{4x^2+2x+1}}{\sqrt{4x^2+2x+1}}=$$

$$=\lim_{x\to\infty}\frac{4(x-1)^4+x^2(4x^2+2x+1)}{\sqrt{4x^2+2x+1}\left[2(x-1)^2-\sqrt{4x^2+2x+1}\right]}=$$

$$=\lim_{x\to\infty}\frac{-18x^3+23x^2-16x+4}{-x^3\sqrt{4+2\frac{1}{x}+\frac{1}{x^2}}*\left[2\left(1-\frac{1}{x}\right)^2+\sqrt{4+2\frac{1}{x}+\frac{1}{x^2}}\right]}=\frac{9}{4}$$

$\Rightarrow y = -x + \dfrac{9}{4}$ este asimptota oblica pentru $x \to -\infty$

IV. $f`(x) = \dfrac{2(x-1)(4x^2+7x+3)}{(4x^2+2x+1)^{3/2}}$

$$f`(x) = 0 \Rightarrow 2(x-1)(4x^2+7x+3) = 0 \Rightarrow x`_1 = 1;\ \ x`_2 = -1;\ \ x`_3 = -\frac{3}{4}$$

$$f(1) = 0;\ \ f(-1) = \frac{8\sqrt{3}}{3} = 4{,}6188;\ \ f(-\frac{3}{4}) = \frac{7\sqrt{7}}{4} = 4{,}63$$

V. $f``(x) = \dfrac{2(47x^2+46x+5)}{(4x^2+2x+1)^{5/2}}$

$$f``(x) = 0 \Rightarrow 47x^2+46x+5 = 0 \Rightarrow x = \frac{-23 \pm 7\sqrt{6}}{47} \Rightarrow$$

$\Rightarrow x``_1 = -0{,}854;\ \ \ x``_2 = -0{,}125$

$$f(x``_1) = 4{,}625;\ \ \ f(x``_2) = 2{,}805$$

V. Tabloul de variatie:

x	-∞ -1 -0,854 -3/4 -0,125 0 1 ∞
f `(x)	- - - 0 + + + + 0 - - - - - - 0 + +
f ``(x)	+ + + + 0 - - - - - 0 + + + + +
f(x)	+∞ 4,619 4,625 4,630 2,805 2 0 +∞

4. Sa se reprezinte grafic "*Serpentina lui Newton*" data prin functia:

$$f(x) = \frac{ax}{ax^2 + 1} \ , \ a > 0$$

Demonstratie :

I. a) $f : R \to R$

b) $G_f \cap Ox : f(x) = 0 \Rightarrow \frac{ax}{ax^2 + 1} = 0 \Rightarrow ax = 0 \Rightarrow x = 0 \ : O(0,0)$

c) $G_f \cap Oy : f(0) = 0 \Rightarrow O(0,0)$

d) $\lim\limits_{x \to \pm\infty} f(x) = \lim\limits_{x \to \pm\infty} \frac{ax}{ax^2 + 1} = 0$

II. $f(-x) = -\frac{ax}{ax^2 + 1} = -f(x)$

$\Rightarrow f$ este functie impara, deci G_f este simetric fata de origine.

III. asimptote orizontale : y = 0

asimptote oblice : ____

asimptote verticale : ____

IV. $f`(x) = \left(\frac{ax}{ax^2 + 1}\right)` = \frac{a(ax^2 + 1) - 2a^2x^2}{(ax^2 + 1)^2} = \frac{-a^2x^2 + 1}{(ax^2 + 1)^2} = a\frac{-ax^2 + 1}{(ax^2 + 1)^2}$

$f`(x) = 0 \Rightarrow a\frac{-ax^2 + 1}{(ax^2 + 1)^2} = 0 \Rightarrow -ax^2 + 1 = 0 \Rightarrow x = \pm\frac{1}{\sqrt{a}}$

$f(\frac{1}{\sqrt{a}}) = \frac{\sqrt{a}}{2}; \quad f(-\frac{1}{\sqrt{a}}) = -\frac{\sqrt{a}}{2}$

V. $f``(x) = \left(\frac{-a^2x^2 + 1}{(ax^2 + 1)^2}\right)` = \frac{-2a^2x(ax^2 + 1)^2 - 4ax(-a^2x^2 + 1)(ax^2 + 1)}{(ax^2 + 1)^4} = \frac{2a^2x(a^2x^4 - 2ax^2 - 3)}{(ax^2 + 1)^4} =$

$= \frac{2a^2x(ax^2 + 1)(ax^2 - 3)}{(ax^2 + 1)^4} = \frac{2a^2x(ax^2 - 3)}{(ax^2 + 1)^3}$

$f``(x) = 0 \Rightarrow 2a^2x(ax^2 - 3) = 0 \Rightarrow x_1 = 0; \quad x_{2,3} = \pm\sqrt{\frac{3}{a}}; \quad f\left(\sqrt{\frac{3}{a}}\right) = \frac{\sqrt{3a}}{4}; \quad f\left(-\sqrt{\frac{3}{a}}\right) = -\frac{\sqrt{3a}}{4}$

x	-∞		- 3/a		- 1/ a		0		1/ a		3/a		+∞
f \`(x)	-	-	-	-	0	+	+	+	0	-	-	-	-
f \`\`(x)	-	-	0	+	+	+	0	-	-	-	0	+	+
f(x)	0		- 3a /4		- a /2		0		a /2		3a /4		0

5. Sa se reprezinte grafic functia:

$$f(x) = \sqrt[3]{x^2} \pm \sqrt{a^2 - x^2} \quad \text{(CARDIOIDA)}$$

I. a) $a^2 - x^2 \geq 0 \Leftrightarrow x^2 \leq a^2 \Leftrightarrow x \leq |a| \Rightarrow x \in [-|a|, |a|]$

$f : [-|a|, |a|] \to R$

$$f(x) = \begin{cases} \sqrt[3]{x^2} + \sqrt{a^2 - x^2} \\ \sqrt[3]{x^2} - \sqrt{a^2 - x^2} \end{cases}$$

$f_1(x) = \sqrt[3]{x^2} + \sqrt{a^2 - x^2}$

$f_2(x) = \sqrt[3]{x^2} - \sqrt{a^2 - x^2}$

b) $G_f \cap Ox:\ f_1(x) = 0;\ \sqrt[3]{x^2} + \sqrt{a^2 - x^2} > 0$

$\Rightarrow G_{f_1}$ nu intersecteaza axa Ox; se afla in intregime deasupra axei absciselor

$f_2(x) = 0;\ \sqrt[3]{x^2} - \sqrt{a^2 - x^2} = 0 \Leftrightarrow \sqrt[3]{x^2} = \sqrt{a^2 - x^2} \uparrow^6 \Leftrightarrow$

$\Leftrightarrow x^4 = (a^2 - x^2)^3 \Leftrightarrow x^4 = a^6 - 2a^4x^2 + a^2x^4 - a^4x^2 + 2a^2x^4 - x^6$

$\Leftrightarrow x^6 - x^4(3a^2 - 1) + 3a^4x^2 - a^6 = 0$

Fie $g(x) = x^6 - x^4(3a^2 - 1) + 3a^4x^2 - a^6$

$$\left.\begin{array}{l} g(0) = -a^6 \\ g(\pm a) = a^4 \end{array}\right\} \Rightarrow g(0)g(\pm a) = (-a^6)a^4 < 0 \xrightarrow{\text{lema intersectiei}} (\exists) \text{cel putin un } x \text{ astfe incat } g(x) = 0$$

$\Rightarrow x_1 \in (0, |a|);\ x_2 \in (-|a|, 0)$

c) $G_f \cap Oy:\ f_1(0) = |a|;\quad f_2(0) = -|a|$

$f(\pm a) = f_1(\pm a) = f_2(\pm a) = \sqrt[3]{a^2}$

d) $\lim\limits_{\substack{x \to |a| \\ x \leq |a|}} f(x) = \sqrt[3]{a^2}$

$\lim\limits_{\substack{x \to -|a| \\ x \geq -|a|}} f(x) = \sqrt[3]{a^2}$

II. $f(-x) = f(x) \Rightarrow$ functia este para, deci graficul asociat este simetric fata de axa ordonatelor

III. asimptote orizontale : ___

asimptote oblice : ___

asimptote verticale : ___

IV. $f`(x)=\frac{2}{3}\frac{1}{\sqrt[3]{x}}\mp\frac{x}{\sqrt{a^2-x^2}}$

$f_1`(x)=\frac{2}{3}\frac{1}{\sqrt[3]{x}}-\frac{x}{\sqrt{a^2-x^2}}$

$f_2`(x)=\frac{2}{3}\frac{1}{\sqrt[3]{x}}+\frac{x}{\sqrt{a^2-x^2}}$

$f`(x)=0 \Leftrightarrow f_1`(x)=0;\ \ f_2`(x)=0$

- pentru $x\in(0,|a|)$ $f_2`(x)>0 \Rightarrow f_2$ este strict crescatoare pe $(0,|a|)$
- pentru $x\in(-|a|,0)$ $f_1`(x)<0 \Rightarrow f_2$ este strict descrescatoare pe$(-|a|,0)$

$f_1`(x)=0 \Leftrightarrow 2\sqrt{a^2-x^2}-3x\sqrt[3]{x}=0 \Leftrightarrow 2\sqrt{a^2-x^2}=3x\sqrt[3]{x}\uparrow^6$

$\Rightarrow 729x^8+64x^6-192a^2x^4+192a^4x^2-64a^6=0$

fie $h(x)=729x^8+64x^6-192a^2x^4+192a^4x^2-64a^6$

$\left.\begin{array}{l}h(0)=-64a^6\\ h(\pm a)=729a^8\end{array}\right\}\Rightarrow h(0)h(\pm a)=(-64a^6)729a^8<0\xrightarrow{\text{lema intersectiei}}(\exists)x`_1\in(0,|a|)\text{ si }x`_2\in(-|a|,0)$

astfel incat $f_1`(x`)=0 \Rightarrow x`_1$ si $x`_2$ sunt puncte de extrem

$\lim\limits_{\substack{x\to 0\\ x>0}} f`(x)=+\infty$ si $\lim\limits_{\substack{x\to 0\\ x<0}} f`(x)=-\infty$ $\Rightarrow$ punctele de coordonate: $(0, f(0))$, adica

$(0,|a|)$ si $(-|a|,0)$ sunt puncte unghiulare

Deoarece $\lim\limits_{\substack{x\to |a|\\ x<|a|}} f`(x)=-\infty$ si $\lim\limits_{\substack{x\to -|a|\\ x>-|a|}} f`(x)=+\infty$ atunci G_{f_1} este tangent la dreptele de ecuatie: $x=\pm|a|$

V. $f``(x)=-\frac{2}{9}\ \frac{1}{\sqrt[3]{x^4}}\mp\frac{a^2}{(a^2-x^2)^{3/2}}$

$f_1``(x)=-\frac{2}{9}\ \frac{1}{\sqrt[3]{x^4}}-\frac{a^2}{(a^2-x^2)^{3/2}}$

$f_2``(x)=-\frac{2}{9}\ \frac{1}{\sqrt[3]{x^4}}+\frac{a^2}{(a^2-x^2)^{3/2}}$

– pentru $x\in[-|a|,|a|]$ $f_1``(x)<0 \Rightarrow$ ramura f_1 este concava.

– pentru $f_2``(x)=0 \Rightarrow$

$$-\frac{2}{9}\,\frac{1}{\sqrt[3]{x^4}}+\frac{a^2}{(a^2-x^2)^{3/2}}=0 \Leftrightarrow \frac{2}{9}\,\frac{1}{\sqrt[3]{x^4}}=\frac{a^2}{(a^2-x^2)^{3/2}} \quad \uparrow^6$$

$$\Rightarrow 64(a^2-x^2)^9-9^6a^{12}x^8=0$$

Fie $j(x)=64(a^2-x^2)^9-9^6a^{12}x^8$

$$\left.\begin{array}{l} j(0)=64a^{18} \\ j(\pm a)=-9^6a^{20} \end{array}\right\} \Rightarrow j(0)\,j(\pm a)=(-9^6a^{20})64a^{18}<0 \to (\exists)x``_2 \in (-|a|,0) \text{ si } x``_1 \in (0,|a|)$$

astfel incat $f_2``(x)=0 \Rightarrow x``_1$ si $x``_2$ puncte de inflexiune

VI. Tabloul de variatie al functiei se face separat pentru cele doua ramuri:

x	-$\lvert a\rvert$ x\`2 0 x\`1 $\lvert a\rvert$
f \`1(x)	+ + + 0 - - $\vert$ + + 0 - - - -
f \`\`1(x)	- - - - - - - - - - - - -
f 1(x)	a $\lvert a\rvert$ a

x	-$\lvert a\rvert$ x\`\`2 0 x\`\`1 $\lvert a\rvert$
f \`2(x)	- - - - $\vert$ + + + +
f \`\`2(x)	+ + 0 - - $\vert$ - - 0 + +
f2(x)	a -$\lvert a\rvert$ a